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YI ZHENG

COMPETITION AND COORDINATION

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Abstract

This thesis discusses competition and coordination in the market. On the supply side, firms decide optimal competing strategies in pricing while considering corporate social responsibility. On the demand side, on one hand, consumers can act selfishly without considering social norms. On the other hand, consumers can coordinate so that better outcomes can be achieved: for example, a reduction in market inefficiency or an improvement in welfare. I analyse the relationship between the demand and supply sides, and study how the preference and behaviour of one side can affect the other. I then discuss the incentives of both firms and consumers, either competitive, non-cooperative or coordinative.

This thesis consists of four articles. The first article points out a new cause of market inefficiency in competitive markets. I propose a coordinative solution to the problem. I show that the market clears if consumers coordinate in a certain way and, therefore, confirm the value of coordination in competitive markets. The second article studies the role of socially responsible actions in a lobbying game. Firms' investment in corporate social responsibility, for example, on environmental protection or animal rights, is proved to be a strategic action that effectively reduces lobbying costs and improves welfare. The other two articles study boycotts. I examine how consumers' behaviour that is driven by environmental concerns can jointly cause a change in firms' behaviour towards more socially responsible actions, and how firms choose their best reactions to deal with boycotts.

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Helsinki, June 2020

Yi Zheng

Contents

Chapter 1

Introduction

In this thesis, I discuss how markets work and how market participants, both firms and consumers, interact with each other and jointly determine market performance. Markets can be viewed as simple supply-demand games or as complex battlefields of market participants. On the supply side, there is competition among firms. On the demand side, there may be coordination among consumers. Between firms and consumers, conflicts may exist. I study these aspects by analyzing the relationship between the demand and supply sides, and discuss how the preferences and behaviour of one side can affect the other. This chapter is organized as follows. I start by discussing the incentives of the firms and consumers and how they interact in the markets in Sections 1.1 to 1.3. I briefly discuss the games that are used in this thesis in Section 1.4. Section 1.5 provides the summary of the four articles. My contribution to the existing literature is highlighted in Section 1.6.

1.1 Price and Corporate Social Responsibility (CSR)

When we discuss firms in competitive markets, we usually think that each firm maximises its profit by competing on the asking prices of the goods (Bertrand competition) or the amount of output they produce (Cournot competition). In this thesis, I focus on price competition. Each firm's optimal strategy in pricing is determined conditional on (the belief of) its competitors' strategies. When none of them have any incentive to change their strategies, they reach Nash equilibrium. The existence of an equilibrium does not imply market efficiency, which refers to the market-clearing situation where the supply is equal to the demand. The optimal strategy refers to an individual firm's profit-maximisation while market efficiency relates to social welfare.

Firms compete to attract more consumers and to obtain a larger market share, thus generating higher profit. Sometimes competitive incentives can have a negative impact on firms' decision-making and does not lead to pleasant results as expected. For example, in wars of attrition, firms compete hard with a periodic cost and eventually realise the winning prize is far lower than the cumulative costs over the periods of competition. They could have done better by stopping immediately in the very first period. I discuss more about wars of attrition and all pay auctions in Chapters 3 and 4.

Nowadays, firms may find it difficult to survive in the market if they are purely profit-driven. There are more issues that need to be taken into account: environmental protection, employees' demands for fair wages and proper working conditions, human rights, animal rights and gender equality, just to list a few. A firm that neglects these issues may suffer damage to its corporate image and reputation, as well as the financial loss arising from public protests (petitions or threats to boycott) or boycotts (stopping purchases).

According to MSCI KLD rankings,[1] firms receive scores for being socially responsible if they have committed to or invested in the following issues: (1) environment; (2) community and society; (3) employees and supply chain; (4) customers; and (5) governance and ethics. In this thesis, I mostly use environment as the example to define firms' moral preferences. That is, a firm using polluting production technology is viewed as socially irresponsible; while a firm using environmentally friendly production technology is viewed as socially responsible.

The following aspects go to the heart of the research question in three articles in this thesis. For a firm that voluntarily behaves in a socially responsible way, driven by its moral values, whether costly investment in CSR can generate profitable returns for itself and/or improve welfare? For a firm that does not voluntarily behave in a socially responsible way, what is sufficient to force it to make changes? The force can be driven by, for example, social pressure and boycotts. I discuss these further in Chapters 3 – 5.

[1]MSCI KLD stands for Morgan Stanley Capital International, Kinder, Lydenberg, Domini & Co. For more details about KLD rankings, see `https://www.nbs.net/articles/msci-kld-scores` (accessed on 13 January, 2020).

1.2 Selfishness and Collective Action

Consumers' preferences are a combination of many factors (Alger and Weibull, 2013). It is impossible to say that one consumer is purely selfish, that is, does whatever is best for her/his own interest (homo oeconomicus), or that another consumer is purely moral, that is, does whatever is the 'right' thing based on moral standards (homo kantiensis). Consumers usually have combined preferences of these two, but weigh one more heavily than the other. However, in modelling, for simplifying purposes, we may view some consumers to be the selfish type and others to be the high-moral type.

Consumers can be non-cooperative. They are self-interested without considering, for example, social norms. In mass markets where a punishment is difficult to impose on 'faceless' consumers, communication and commitment become difficult to rely on due to free riding. If they had cooperated in some way, a better outcome could have been achieved. Consider cleaning in a shared flat. If I think one or some of my flatmates will clean the flat at some point, I do not bother to do anything (free riding on the benefits of others' contribution). Unfortunately, free-riding incentives can easily lead to unpleasant results. If everyone thinks alike, there is de facto 'no ride' — no one will clean and therefore there will be no benefit for each resident to free ride on. This tells us that at least one person needs to sacrifice her/his time and energy in order to have a clean house. This can be achieved by cleaning in turn based on an agreed schedule, and punishing the lazy person who does not do her/his shift properly. It can also be achieved by cleaning together so that each is only responsible for a small part. In these cases, the residents are cooperative and the benefits from collective actions are generated. I discuss more about the willingness to sacrifice, the incentives to free ride and the (non-)cooperative behaviour in Chapter 4.

Consumers can be coordinative. They are well-organized and act together effectively towards a common target. The purpose of coordination is to improve social welfare, or at least the total utility of some groups. I discuss the value of coordinative actions in greater detail in Chapter 2.

1.3 Markets

Market participants' incentives and motivations, either competitive, (non-)cooperative or coordinative, jointly determine market performance. The equilibria are thus derived conditional on both firms' and consumers' strategies and preferences.

Consumers can be inactive. This means they are price takers and passively accept the asking prices set by the firms. Firms decide the optimal pricing strategies based on consumers' aggregate demand. On the other hand, consumers can be active. This means they attempt to influence the market and, in some circumstances, are willing to sacrifice their own utility and take costly strategic actions, like in boycotts. Firms may find it necessary to act towards boycotters' wishes, shouldering the attendant costs, in order to attract more consumers.

An equilibrium refers to a stable state in which each participant has no incentive to change her/his/its behaviour. It does not necessarily mean the equilibrium is efficient, for example, such that the market clears. Market clearing refers to the situation in which the goods produced by the firms are all sold to the buyers who demand them and there is nothing left in the market. Market clearing usually links to improvement in social welfare. To some extent, it is a win-win situation for both sides of the market: firms obtain the sales revenue by trades and consumers buy if purchasing generates positive utility, that is, if the valuation of the products is higher than the asking price. I discuss market inefficiency more deeply in Chapter 2.

1.4 Games

Game theory is a tool to analyze market performance and rationalize the behaviour of participants. Several games are used in the thesis: Bertrand competition, all pay auction and war of attrition. The equilibrium solution concepts are Nash and subgame perfect equilibria.

Bertrand competition was named after Bertrand (1883) and later studied and extended by many economists, most notably Edgeworth (1925). It has become one of the very first things we learn in game theory or industrial organization theory courses. A Bertrand competition is a price competition. In a duopoly market, a firm chooses the optimal pricing strategy conditional on (the belief of) its competitor's pricing strategy. A Nash equilibrium is a situation in which both firms set their prices equal to their marginal costs, which brings zero profit to both. As a refinement, a Nash equilibrium is subgame perfect if it is a Nash equilibrium in every subgame of the original game.

Auctions contain interesting features that make them good candidates to model market competition. First, auctions are essentially games of conflicts. Second, bidders are non-cooperative. Third, auctions are usually one-winner games, or in boycotts, one-side games (where either the demand or supply side wins). In this thesis, I do not allow a game to end in a tie. Finally, auctions are usually one-shot games. The loser does not have a chance to replay. In an all pay auction, bidders pay whatever they bid regardless of who the winner is. Like other types of auctions, the one who bids the highest wins. I use an all pay auction to model the competition for a monopoly position between two firms where the socially responsible investments affect the authority's determination of the winner.

A war of attrition is a special case of an all pay auction. The key difference is that a war of attrition is considered as the optimal stopping game, where the one who fights longer wins, and the one who surrenders earlier loses. The game is played once and ends immediately when one player (or one side in a boycott example) stops fighting. Each player pays a cost in every active fighting period until the game ends. The costs spent in the past are sunk costs and therefore do not affect each player's decision-making in the current period. However, the continuation value that will be generated if the game continues to the next period matters.

1.5 Review of the Chapters

In this section, I provide a review of four articles.

1.5.1 Market Inefficiency, Entry Order and Coordination

Chapter 2 is coauthored with Professor Kultti. The causes of market inefficiency are many. We suggest an additional cause — buyers' random entry order. In a market where identical sellers compete for buyers of heterogeneous valuations, first come first served is the norm. Since all buyers choose the cheapest available good, a low-valuation buyer who enters the market late may find the remaining goods unaffordable, which causes markets not to clear. We therefore propose a coordination solution to the market inefficiency problem. We find that in a market where all the high-valuation buyers enter first and all the low-valuation buyers enter afterwards, the market clears effectively. Moreover, we find the inefficiency arising from buyers' entry order becomes less of a problem in larger economies and vanishes in the limit.

1.5.2 Socially Responsible Procurement in Lobbying Game

Chapter 3 is coauthored with Dr Hämäläinen. We study how socially responsible procurement affects the money that firms allocate to influence the outcome of a public procurement contest, modeled as an all-pay auction. Firms can try to increase their competitiveness in the market for government contracts both directly by lobbying and by investing in corporate social responsibility (CSR). CSR investments could take many forms (e.g., corporate governance vs. environmental investments), making them harder to compare for the government authority than lobbying spending. As a result, CSR acts as an effective differentiation strategy for firms. We show that socially responsible procurement (i) alleviates the competition among firms for government contracts, (ii) shifts firms' public relations spending from lobbying to CSR investment, and (iii) decreases the total amount of money that a firm spends to influence the authority. This is welfare-improving in so far that lobbying spending is socially wasteful.

1.5.3 Non-organized Boycott: Alliance Advantage and free-riding Incentives in Uneven Wars of Attrition

Chapter 4 in published in the Eurasian Economic Review. We study non-organized boycott activities. We develop a boycott model in which multiple consumers on the demand side come into conflict with a misbehaving monopolist on the supply side. The goal of the boycott is to force the firm that lacks corporate social responsibility to change its behaviour, for example, abandon polluting production technology in favour of environmentally-friendly actions. We analyze consumers' and firm's incentives and equilibrium strategies. We describe the difficulty of winning a non-organized boycott in reality. We find that consumers' free-riding incentives limit the real boycott power even when the benefits to free ride are small. The larger the market served by the firm, the more likely an individual consumer would stop boycotting (who acts as a strict environmentalist), which leaves fewer boycotters remaining in the costly conflict (who act as loyal supporters of the product). On the other hand, we show that market size does not significantly affect the firm's strategies. For a large firm, the consumer boycott will surely be effective, that is, lead to non-zero boycotter participation, but hardly successful, that is, not lead to the firm's cessation of misbehaviour.

1.5.4 A Note on Firms' Ethics, Consumer Boycotts, and Signalling

Chapter 5 is a note to a published article. In an interesting article, Glazer et al. (2010) develop a duopoly model of consumer boycott to analyze firms' optimal strategies. Firms make decisions on their output and their production technology, either clean or polluting. Boycott refers to non-purchasing action from a polluting firm. Non-boycotting behaviour incurs a cost in social pressure. To avoid such cost, some low-moral consumers join the boycott. The equilibrium is derived under the condition that the low-valuation buyers with sufficiently high demand, that is, those with higher valuation than that of the marginal consumer, would boycott.

In the first part of this article, we suggest that since all the low-valuation consumers are indifferent between two firms (see Lemma 1 in Glazer et al. (2010)), the marginal consumer cannot be uniquely determined. Consequently, there exist many equilibria. We construct equilibria in two polar cases following the settings in the original article. In the second part, we propose a solution to the model by considering Bertrand competition. We find that investing in clean technology (behaving ethically) is not necessarily optimal for firms, in terms of payoffs, although the ethical firms benefit from the consumers' heterogeneous preferences and the cost of social pressure.

1.6 Contribution

In this section, I provide a review of related literature and present my contribution to the topics.

Chapter 2 discusses market inefficiency and buyers' coordination as a potential resolution for this issue. The market inefficiency problem and its many causes are widely studied. The causes are related to externalities or informational problems.(see Radner (1979), Vives (1988, 2017), Angeletos and Pavan (2007, 2009), Gilson and Kraakman (2014)). Adverse selection (Spinnewijn (2017), Einav et al. (2010), Bundorf et al. (2012)) and signalling (Vermaelen (1981), Borenstein et al. (2007)) are prominent examples. We point out a new cause — random entry order of buyers. To the best of our knowledge, there is no literature that focuses directly on market performance where the order in which the participants enter the market is studied. To solve the inefficiency problem, we propose a coordination solution. We find that by coordinating buyers' entry order in a certain way, the market clears. Thus we confirm the value of coordination in the competitive market which is consistent with many studies on congestion games.

The value of coordination has been studied extensively after Wardrop (1952) who first proposed traffic assignment problems and Rosenthal (1973) who first proposed congestion games. The standard solutions to the problems of coordination are taxes or tolls; they are designed to minimise individuals' latency (delay in the traffic) (see, e.g., Caragiannis et al. (2010) and Cole et al. (2006)) or to achieve social optimum (see, e.g., Arnott et al. (1990) and Caragiannis (2013)). Tumer et al. (2009) present two methods to reduce congestion by coordinating the drivers' departure times in order to avoid peak-hour traffic, or by implementing a reward system that penalizes the driver who greedily seeks the lanes with high road capacity. Christodoulou et al. (2009) examine how coordination improves the allocation of scarce and shared resources, for example, commonly used facilities, among selfish players. The quality of coordination is evaluated by the price of anarchy, that is, the ratio between the welfare under the optimal centralised solution and the welfare generated by the worst equilibrium.

In electricity markets, the coordination mechanism has also been studied to reduce transmission congestion, avoid enforced curtailments and ensure the security of energy transmission, see for example Fang and David (1999), Yamina and Shahidehpour (2003a,b), Bjørndal and Jörnsten (2007), Kunz and Zerrahn (2015). (Fang and David, 1999) analyze the transmission congestion problem in an unbundled electric system. To avoid enforced curtailments, the authors propose a coordination solution between electricity users and the Independent System Operator (ISO). That is, in the dispatch periods, the operator uses priority transmission and broadcasts information including prices, line congestion and curtailment so that the users adjust their demands. They also suggest that coordination with a generic operator is best for improving transmission conditions as well as for global social welfare.

These models are usually not market models, in which pricing decisions play an important role. On the contrary, the introduction of markets is often suggested as a solution.

The difficulties of achieving coordination without the help of an outside party can be caused by the agents learning too slowly (Gabuthy et al., 2006), by imperfect information (Bell et al., 2003) or by the large number of participants (Knez and Camerer, 1994). Albrecht (2019) uses a market setting to study a coordination problem. He studies it in (imperfectly) competitive markets which feature Pareto-ranked equilibria. An outcome is deemed a coordination failure if the corresponding equilibrium is not Pareto-optimal. He focuses on the trembling-hand perfect equilibrium, and suggests that efficient equilibria exist in the competitive environments.

We discuss market inefficiency and the value of coordination in a set-up different from all of the above. We consider a market where in principle everyone could trade but where the interaction between pricing and the order of entering the market creates problems. This is a novel cause of inefficiency that is particularly relevant in a small market; when the economy grows the inefficiencies vanish.

Chapter 3 discusses the value of investing in socially responsible projects in lobbying contests. In a competitive market, two firms lobby for the monopoly position by simultaneously choosing the bidding strategies and the investment levels in socially responsible actions. CSR investments are assumed to be preferred by the authority and therefore have an influence on determining the winner[2]. We extend Ellingsen (1991)'s lobbying game by considering the authority's preference for CSR investments and the difficulty of comparing the values and effects of such investments. Our model is similar to Carlin (2009), which analyzes consumer obfuscation in financial products markets, but we replace the price competition model with all-pay auctions, which is the standard way of analyzing contests in the literature. Our results confirm the value of CSR investments consistent with the findings of many game theorists and social

[2]Authorities all over the world have established guidelines for socially responsible procurement. See `https://ec.europa.eu/environment/gpp/index_en.htm` and `https://www.fedcenter.gov/programs/buygreen/index.cfm?` (accessed on 13 January, 2020).

scientists.

The value of CSR has been studied both theoretically and empirically by economists and social scientists. Bénabou and Tirole (2010) suggest that spending on CSR can sometimes be regarded as a firms' adoption of a more long term perspective. Indeed, Kempf and Osthoff (2007) show that socially responsible investing, where companies are selected based on CSR, leads to high abnormal returns. On the other hand, Kotchen and Moon (2012) find empirical support for the hypothesis that companies invest in CSR to offset corporate social irresponsibility. We suggest that CSR is a welfare improving alternative to a more socially costly behavior, i.e., lobbying. Besley and Ghatak (2007) argue that firms do not have a comparative advantage in CSR investment even when consumers desire it because a free-riding problem arises in private provision. Our article differs from earlier work in that we do not focus on the desirability of CSR but rather on its strategic use by firms in socially responsive procurement where the authority regards various investments in CSR favorably; Baron (2001) considers strategic use of CSR by firms in a more general setup where rival firms are targeted by activists. Empirical researches show that firms that invest more in corporate social responsibility (CSR) receive more contracts in procurement (Flammer, 2018) and get a higher return on their lobbying spending (Garcia, 2016).

Our analysis connects (i) the industrial organization literature on differentiation and obfuscation and (ii) the public choice literature that analyzes rent-seeking in lobbying contests. Investing in CSR is an efficient lobbying strategy in our model because it enables a firm to differentiate from its rival and make the choice among firms harder for the authority.[3] This is shown to relax the competition for government contracts similarly to what happens in markets; see Perloff and Salop (1985), Wolinsky (1986), Shaked and Sutton (1982). There is also a link to the literature on strategic complexity and obfuscation in consumer markets, e.g., Gabaix and Laibson (2006), Ellison and Wolitzky (2012), Wilson (2010), Piccione and Spiegler (2012), Chioveanu and Zhou (2013), which observe that firms have incentives to make their products harder for consumers to analyse. This has adverse effects on consumers and market welfare.[4] However, our article shows that in contests the effects of CSR investment strategy could instead be positive since by alleviating competition it additionally reduces wasteful

[3]For a classic article on environmental product differentiation, see Reinhardt (1998).
[4]But see Taylor (2017) where obfuscation allows screening and improves welfare.

spending. This is because competition is not usually productive in lobbying, unlike in markets.[5]

The last two chapters discuss boycotts, that is, conflicts between firms lacking awareness of corporate social responsibility and consumers with moral concerns. **Chapter 4** studies non-organized boycott activities in a monopoly market where two consumers boycott the polluting firm (a two-against-one game). The model extends Maynard Smith (1974)'s classic two-contestant game of war of attrition by adding the third player to the demand side. By introducing free-riding incentives and alliance advantages to the demanders, we have a game where two consumers play a prisoners' dilemma against each other and jointly they play a war of attrition against the polluting firm. Limited studies have focused on a third contestant's contribution and influence in a war of attrition framework taking account of incentives to free ride and alliance advantage. The closest studies to ours are Haigh and Cannings (1989); Bulow and Klemperer (1999) and Helgesson and Wennberg (2015), which discuss n-player competing for one or several prizes and Powell (2017) discusses third-party intervention in wars.

Several articles study consumer boycotts in different settings. Friedman (1991); Delacote (2008) provide conceptual discussion on boycott actions. Tyran and Engelmann (2005) provide an experiment on boycott in reaction to a sudden cost increase in retail markets. They find that the cost increases the incidence of boycotts. Boycotts reduce market efficiency. Innes (2006) develops a model where two non-identical duopolists face a threat to boycott from an environmental organisation. He finds that at equilibrium a small persistent boycott against the small firm or a large transitory boycott would work against the large firm. It implies that larger firms are easier to defeat. Baron (2001) employs a game between an influential activist and a monopolist that has concerns for profit maximisation, altruism and activist's powerful threats. From a psychological perspective, John and Klein (2003) explain consumers' boycotting incentives and willingness to sacrifice. Heijnen and van der Made (2012) find that in a market under asymmetric information where consumers can signal high moral values, consumers always boycott with positive probability despite free-riding incentives. It will eventually result in a change in the firm's behaviour. In a war of attrition framework, Peck

[5] As known since Tullock (1967), the possibility of acquiring a monopoly position by influencing public choice not only reduces welfare by alleviating competition in product markets (captured by the "Harberger triangle" of deadweight loss) but also generates losses because of unproductive lobbying competition (represented by the "Tullock square" of dissipated rents).

(2017) analyzes a game between a monopolist that produces two-period durable goods and consumers that demand a lower price. He derives both non-boycott equilibrium and boycott equilibria where boycott occurs with positive probability. Egorov and Harstad (2017) develop a boycott game between a public regulator, a misbehaving firm and activists. They find that in a two-player game without the regulator, 'private politics' is beneficial for activists but harmful for firms. Meanwhile, in a three-player game, 'private politics' is harmful for activists but beneficial for firms. I contribute to the literature by demonstrating that the small benefit of free-riding is sufficient to undermine the probability of boycott success. I also discuss how market size affects a firm's decision-making. Therefore I describe the difficulty of winning a non-organized boycott in reality and give motivation for further research on well-organized boycotts.

Chapter 5 serves as a discussion note and an extension to a published article by Glazer et al. (2010) which studies boycotts in a duopoly market. I first discuss the determination of the equilibrium in the original article. I suggest the existence of a continuous set of equilibria. Then I propose a solution to the model by considering Bertrand competition.

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Chapter 2

Market Inefficiency, Entry Order and Coordination[1]

2.1 Introduction

The standard causes of market inefficiency are related to externalities or informational problems that manifest in the pay-off relevant private information of some market participants (see Radner (1979), Vives (1988, 2017), Angeletos and Pavan (2007, 2009), Gilson and Kraakman (2014)). Adverse selection (Spinnewijn (2017), Einav et al. (2010), Bundorf et al. (2012)) and signalling (Vermaelen (1981), Borenstein et al. (2007)) are prominent examples. In this article we point out an additional phenomenon that give rise to inefficient outcomes. It is the order in which market participants enter the market. When there are, say, buyers with heterogeneous valuations, and capacity-constrained sellers who price the goods before the buyers enter, the sellers typically use mixed strategies in pricing. The low pricing sellers target low-valuation buyers, and make sure that they get to trade. Other sellers take some risk and target high-valuation buyers. The risk arises as buyers always buy the cheapest available goods. If high-valuation buyers enter first there are only high-priced goods left to the low-valuation buyers; not all possible trades are consummated.

Consider an example: in an online dating network, women and men search for partners. Women post the selection criteria of desired men such as hobbies, job, education, height, age and so on. Men read the posts and contact the women if they

[1]This chapter is based on an article jointly written with Klaus Kultti.

meet the criteria. The criteria are viewed as an implicit prices. Women and men are viewed as individual sellers and buyers, respectively. Of course women are different from each other, but for simplicity we assume they are identical. We also assume that once a woman receives a contact, she leaves the dating network immediately so that there is no further searching and matching. Therefore, in this dating network, the matching results depend mainly on the price level (the posted selection criteria) and buyers' timing of entering the market (the time that men start searching online and finding the posts). Entering the dating network too late results in a lower chance of finding a match since some women have already left the market by then.

This simple observation can be used to study the value of coordination in markets where competition amongst the sellers does not solve the problem. We employ a setting where there are equal numbers of sellers and buyers. The sellers are identical with one unit of an indivisible good for sale. They set the prices prior to the arrival of the buyers and commit to the sticky prices. The buyers are of two types. Half of them are low-valuation buyers whose reservation price is v, and the other half are high-valuation buyers with reservation price unity. Each buyer has unit demand for the good.

We employ a static game. Once the prices are posted, the buyers have take-it-or-leave-it offers and price negotiation is not allowed. The reason for such a setting is as follows. In a dynamic setting where prices can be adjusted, that is, the sellers can lower the prices to serve the low-valuation consumers, the market will clear and the inefficiency problem will be solved. However, we need to take into account that the high-valuation buyers have an incentive to pretend to be of low-valuation so that they obtain the goods with lower prices. The discount factor also affects the timing of purchasing. These features complicate the analysis and are not helpful in solving the inefficiency problem of our interests.

We analyze three different scenarios. In the benchmark case, the buyers enter the market in a random order, and in a symmetric equilibrium the sellers use mixed strategies in pricing. The remaining cases constitute the two polar ways of coordinating the buyers' order of entry to the markets. In one case all the high-valuation buyers enter the market first. It turns out that the sellers' symmetric pricing strategy is a pure one where every seller asks price v. All the possible trades are consummated, and there are no inefficiencies. In the other case, all the low-valuation buyers enter the market first. The equilibrium pricing is in mixed strategies, and competition for the high-valuation buyers is more intense than in the benchmark case. As a result, the allocation is also more inefficient.

What is notable is that in all three cases the sellers' expected pay-off is v, and in this sense the value of coordination can be evaluated by considering the buyers only; the sellers do not care what the buyers do. It is worth noting that the value of coordination does not arise from there being more resources available nor there being equilibria that can be Pareto-ranked; in all the cases there is exactly one symmetric equilibrium. The inefficiencies arise because of the order in which the buyers enter the market, and for efficiency comparison, the ordering is the only thing we vary.

We follow Satterthwaite and Williams (1989) and measure the market (in)efficiency using the ratio between the total ex-ante value created and the total ex-ante value created if the market clears. In the benchmark case we attain explicit expressions for the inefficiency. The results demonstrate that the inefficiency vanishes quite quickly as the economy grows. The case where the low-valuation buyers enter the market first is more complicated in terms of explicit expressions but we provide an approximate solution and conduct numerical analysis. We find that the inefficiencies are much more substantive, and vanish much more slowly than in the benchmark case as the economy grows. In all cases, however, the degree of inefficiency becomes smaller as the economy becomes larger, and vanishes in the limit.

We are not aware of literature that focuses directly on market performance where the order in which the participants enter the market is studied. Of course, the value of coordination is recognised in a multitude of settings. For instance, almost by definition, any model of congestion demonstrates the value of coordination. In traffic settings, taxes and tolls are proposed in Caragiannis et al. (2010) and Cole et al. (2006).

There are many reasons why achieving coordination is difficult without an outside party but the number of participants is clearly one of the most important. Knez and Camerer (1994) suggest that coordination succeeds in a two-player game but difficulties arise in a group of three or more players. Albrecht (2019) uses a market setting to study a coordination problem. He studies it in a (imperfectly) competitive matching market with sunk investments which features Pareto-ranked equilibria. An outcome is deemed a coordination failure if the corresponding equilibrium is not Pareto-optimal.

He shows that with sufficient heterogeneity of the participants all the equilibria are efficient once the solution concept is refined to trembling-hand perfectness.

Our article discusses market inefficiency and the value of coordination in a set-up different from all of the above. We consider a market where in principle everyone could trade but where the interaction between pricing and the order of entering the market creates problems. This is a novel cause of inefficiency that is particularly relevant in a small market. When the economy grows the inefficiency vanishes, which is contrary to what Knez and Camerer (1994) observe in their experiments.

We find that a market setting without any inherent frictions, like physically separated sellers, exhibits inefficiencies that are not related to private information. A natural assumption that the buyers enter the market in a random order, associated with the sellers' capacity constraints gives rise to strategic behaviour in pricing leading to unattainable gains from trade. In general, one would expect that if the markets are small, the price taking assumption does not hold, and there is a case for strategic behaviour. One would also expect that as the market grows, strategic behaviour becomes less important.

If one party, the buyers, coordinates its actions, the other party, the sellers, responds by changing its pricing behaviour. The sellers are always able to change their behaviour in a way that retains their expected pay-off regardless of the buyers' actions. In a large economy, the resources per capita remain the same, and consequently the gains in efficiency are due to the changes in the sellers' actions. A redeeming feature of the model is that the gain from strategic behaviour vanishes, as well as the inefficiencies. We point out a specific way to coordinate the buyers' order of entry that solves the problem, and we also study the other extreme, namely the entry order of the buyers that generates the most inefficient outcome. This is interesting because it results in the fiercest competition amongst the sellers in the sense that fewer trades are consummated and more resources are wasted. It demonstrates that competition as such does not lead to an efficient outcome.

The rest of the article is organized as follows. In Section 2.2, we develop a benchmark model where buyers enter the market in a random order. We derive the equilibrium strategy for sellers and analyze the market efficiency. In Section 2.3, we study the coordination model where buyers of same type enter the market at the same time. In

Section 2.4, we provide the numerical solutions for the equilibrium strategies and the measures for market efficiency. We compare the results of the benchmark and the coordination games and discuss how market size, buyers' valuation and coordinative actions affect market efficiency. In Section 2.5, we discuss the logic of how the coordination would happen in the market so that it could effectively solve the market inefficiency problem. Section 2.6 concludes the article.

2.2 Benchmark Model: Random Entry Order

Consider an economy where there are $2n$ sellers, each with a unit of an indivisible good, serving $2n$ buyers.[2] The good is not long-lasting so the unsold items cannot be put into the resale market with discounted prices. The sellers are identical while the buyers are of two types. Half of the buyers value the good at unity and the other half at $v < 1$. All these are common knowledge. The game is static and in two stages. In Stage One, the sellers make the pricing decisions prior to the arrival of the buyers. Once the prices are posted, the sellers do not adjust them.[3] In Stage Two, the buyers enter the market in a random order. The choice set for each buyer is binary: she/he buys if her/his valuation is higher than the lowest available price; Otherwise she/he does not buy. All the sellers are in the same location so that the buyers can see all the prices and then choose the lowest-priced good.

We impose the following assumption.

Assumption 1. *The valuation of the low-type buyers v is less than* $1/2$.

If v is higher, the equilibrium is a pure strategy one where each sellers asks price v. Assumption 1 guarantees that the pricing is in mixed strategies. Note that we could also guarantee a mixed strategy equilibrium by varying the proportion of high-valuation buyers but it is simpler to focus on v.

To derive the sellers' pricing strategies, we first show that there is no pure strategy.

[2] If there are more sellers than buyers, a Bertrand-outcome where sellers reduce the price to zero ensues. If there are more buyers than sellers, sellers will set high prices to target the high-valuation buyers and leave some (or all) low-valuation buyers unserved. The profit will attract more sellers to enter the market. Equal numbers of buyers and sellers would be the outcome if we had an entry stage with entry cost $c < v$.

[3] The logic of the sellers' incentive to commit to the sticky prices is as follows: if the price is not sticky but can be negotiated to serve buyers of heterogeneous types, the buyers have incentive to pretend to belong to the low-valuation type.

Lemma 1. *The sellers use symmetric mixed strategies in pricing.*

Proof. We prove by contradiction. Suppose there is a pure strategy such that identical sellers set the price at p'. First, we show that $p' = 1$ or $p' \in [0, v]$ cannot be an equilibrium. When $p' = 1$, the probability of a trade is the probability of matching with a high-valuation buyer which is 1/2. Therefore the sellers' expected profit is 1/2. If one deviates to a slightly lower price $1 - \epsilon$ where ϵ is a small positive number, it leads to a trade with one of the high-valuation buyers surely and thus generates the utility $1 - \epsilon$. That is, for any $\epsilon < 1/2$, it is a profitable deviation. Similarly $p' = v$ cannot be an equilibrium. Asking price v would sell the good with certainty. A deviation to price 1 generates the expected utility 1/2, which is higher than the utility of asking v by Assumption 1. For the same reason, any price that is lower than v clearly cannot be an equilibrium.

Finally, consider price $p' \in (v, 1)$. Any price in this range is too expensive for the low-valuation buyers. Therefore $2n$ goods are on offer to n high-valuation buyers. It leaves each seller the expected utility $p'/2$. A deviation to a slightly lower price $p' - \epsilon$ ensures the sell and generates a profit $p' - \epsilon$. That is, for any $\epsilon < p'/2$, there is a profitable deviation. This completes the proof. □

Next, we derive a symmetric mixed strategy equilibrium. It is clear that the sellers use mixed strategies such that with probability $\rho^A_{(n)}$ they post price v, and with probability $1 - \rho^A_{(n)}$ they use a continuous mixed pricing strategy G on some interval $[a, 1]$ where $v < a < 1$ and the value of $\rho^A_{(n)}$ depends on the market size, that is, the exogenous value of n. The mixed strategy implies that the sellers target consumers of both types. The probability $\rho^A_{(n)}$ cannot be zero. Otherwise $2n$ sellers compete for n high-valuation buyers and leave all the low-valuation buyers unserved. The outcome is similar as in a Bertrand competition where sellers reduce the price to v.

To evaluate the performance of the market, we first solve the number of unsold items and the probabilities that such circumstances occur by taking account of the buyers' random entry order. Denote the number of sellers who ask price v by j. We find the following result.

Lemma 2. *The expected number of unsold goods is* $n(1 - \rho^A_{(n)})$.

Proof. We first show that if fewer than half of the sellers ask price v, that is, if $j < n$, the number of unsold goods varies between $n - j$ and n; if at least half of the sellers ask price v, that is, if $j \geq n$, the number of unsold goods varies between zero and $2n - j$.

When $j < n$ and the low-valuation buyers enter the market first, they buy all the goods priced at v and find the remaining goods unaffordable. The high-valuation buyers then enter the market and buy the cheapest available goods. That is, j goods go to the low-valuation buyers and n go to the high valuation, which leaves $n - j$ unsold. If the high-valuation buyers enter the market first, they buy j cheap and $n - j$ expensive goods. Since all the n remaining goods are too expensive for the low-valuation buyers, none is sold. Therefore the number of unsold items varies between $n - j$ and n for $j < n$.

The probability that the number of unsold items is $n - j + k$, $k \in \{0, 1, ..., j\}$, is given by $Pr\left(n - j + k\right) = \frac{\binom{n}{j-k}\binom{n}{k}}{\binom{2n}{j}}$. In the denominator there is the number of ways to choose j from a total of $2n$ sellers in the market. In the numerator there is the product of choosing $j - k$ low-valuation buyers and k high-valuation buyers among the first j buyers.

Analogously, when $j \geq n$, the number of unsold goods varies between zero, if the low-valuation buyers enter the market first, and $2n - j$, if the high-valuation buyers enter first. The probabilities are given by $Pr\left(k\right) = \frac{\binom{n}{j-(n-k)}\binom{n}{n-k}}{\binom{2n}{j}}$, $k \in \{0, 1, ..., 2n - j\}$.

The expected number of unsold goods, when $j < n$, is given by

$$\sum_{k=0}^{j} \frac{\binom{n}{j-k}\binom{n}{k}}{\binom{2n}{j}} (n - j + k). \tag{2.1}$$

This event happens with probability $\binom{2n}{j}\left(\rho^A_{(n)}\right)^j\left(1 - \rho^A_{(n)}\right)^{2n-j}$. Consequently, the ex-ante expectation of the number of unsold goods is given by

$$\begin{aligned}
&\sum_{k=0}^{j} \frac{\binom{n}{j-k}\binom{n}{k}}{\binom{2n}{j}} (n - j + k) \binom{2n}{j} \left(\rho^A_{(n)}\right)^j \left(1 - \rho^A_{(n)}\right)^{2n-j} \\
&= \sum_{k=0}^{j} \binom{n-1}{j-k}\binom{n}{k} (n - j + k) \left(\rho^A_{(n)}\right)^j \left(1 - \rho^A_{(n)}\right)^{2n-j} \\
&= \sum_{k=0}^{j} n \binom{n-1}{j-k}\binom{n}{k} \left(\rho^A_{(n)}\right)^j \left(1 - \rho^A_{(n)}\right)^{2n-j}
\end{aligned}$$

where $\rho^A_{(n)}$ is the probability that sellers set the price at v. By Vandermonde's identity, we have $\sum_{k=0}^{j}\binom{n-1}{j-k}\binom{n}{k}=\binom{2n-1}{j}$. Therefore the last equality is simplified to

$$\binom{2n-1}{j}n\left(\rho^A_{(n)}\right)^j\left(1-\rho^A_{(n)}\right)^{2n-j}. \tag{2.2}$$

When $j\geq n$, the expected number of unsold goods is given by

$$\sum_{k=1}^{2n-j}\frac{\binom{n}{j-(n-k)}\binom{n}{n-k}}{\binom{2n}{j}}k. \tag{2.3}$$

This event happens with probability $\binom{2n}{j}\left(\rho^A_{(n)}\right)^j\left(1-\rho^A_{(n)}\right)^{2n-j}$. Consequently, the ex-ante expectation of the number of unsold goods is given by

$$\begin{aligned}
&\sum_{k=1}^{2n-j}\frac{\binom{n}{j-(n-k)}\binom{n}{n-k}}{\binom{2n}{j}}k\binom{2n}{j}\left(\rho^A_{(n)}\right)^j\left(1-\rho^A_{(n)}\right)^{2n-j}\\
&=\sum_{k=1}^{2n-j}\binom{n}{j-(n-k)}\binom{n}{n-k}k\left(\rho^A_{(n)}\right)^j\left(1-\rho^A_{(n)}\right)^{2n-j}\\
&=\sum_{k=1}^{2n-j}n\binom{n}{2n-j-k}\binom{n-1}{k-1}\left(\rho^A_{(n)}\right)^j\left(1-\rho^A_{(n)}\right)^{2n-j} \quad \text{by } \binom{n}{j-(n-k)}=\binom{n}{2n-j-k}\\
&=\sum_{k=0}^{2n-j-1}n\binom{n}{2n-j-1-k}\binom{n-1}{k}\left(\rho^A_{(n)}\right)^j\left(1-\rho^A_{(n)}\right)^{2n-j}.
\end{aligned}$$

Again, by Vandermonde's identity, we have $\sum_{k=0}^{2n-j-1}\binom{n}{2n-j-1-k}\binom{n-1}{k}=\binom{2n-1}{2n-1-j}$. Therefore the last equation is equivalent to

$$n\binom{2n-1}{2n-1-j}\left(\rho^A_{(n)}\right)^j\left(1-\rho^A_{(n)}\right)^{2n-j}. \tag{2.4}$$

From Expressions (2.2) and (2.4), the expected number of unsold goods for any j is given by

$$\begin{aligned}
&\sum_{j=0}^{n-1}n\binom{2n-1}{j}\left(\rho^A_{(n)}\right)^j\left(1-\rho^A_{(n)}\right)^{2n-j}+\sum_{j=n}^{2n-1}n\binom{2n-1}{2n-j-1}\left(\rho^A_{(n)}\right)^j\left(1-\rho^A_{(n)}\right)^{2n-j}\\
&=n\left(1-\rho^A_{(n)}\right)\sum_{j=0}^{n-1}\binom{2n-1}{j}\left(\rho^A_{(n)}\right)^j\left(1-\rho^A_{(n)}\right)^{2n-1-j}\\
&\quad+n\left(1-\rho^A_{(n)}\right)\sum_{j=n}^{2n-1}\binom{2n-1}{j}\left(\rho^A_{(n)}\right)^j\left(1-\rho^A_{(n)}\right)^{2n-1-j}.
\end{aligned} \tag{2.5}$$

Denote the Binomial distribution function of $Bin(p,n)$ by $F(p,n,x)$. The above expression is equivalent to

$$n\left(1-\rho^A_{(n)}\right)\left[F\left(\rho^A_{(n)},2n-1,n-1\right)+\left(1-F\left(\rho^A_{(n)},2n-1,n-1\right)\right)\right]$$

which reduces to $n\left(1-\rho^A_{(n)}\right)$. This completes the proof of Lemma 2. □

Next, we determine $\rho^A_{(n)}$, that is, the probability that the sellers ask price v. We already know that the pricing is in mixed strategies. The prices are either at v or continuous in the range of $[a,1]$ where $v<a<1$. At equilibrium, a seller's utility of asking price v, which is the lowest price in the support of the mixed strategy, must equal the utility of asking price 1, which is the highest in the support.

Denote a seller's linear utility function by $U(p)$ where p is the price asked by the seller.[4] If a seller asks price v, since every buyer has the same or higher valuation for the good, the seller trades for certain. Therefore the seller's expected utility is

$$U(v)=v\cdot 1=v. \tag{2.6}$$

If a seller asks the highest price unity, we need to consider two cases. If fewer than half of sellers set their price at v, that is, when $j<n$, the seller does not trade regardless of buyers' entry order. Unity would be too expensive for the low-valuation buyers and high-valuation buyers choose lower-priced goods. If at least half of the sellers set price at v, that is, when $j\geq n$, buyers' entry order determines the trading probability. Let the indicators $\mathbb{1}_{\{j<n\}}$ and $\mathbb{1}_{\{j\geq n\}}$ represent the two cases discussed above. The expected utility of asking price 1 is

$$U(1)=\sum_{j=0}^{2n-1}\binom{2n-1}{j}\left(\rho^A_{(n)}\right)^j\left(1-\rho^A_{(n)}\right)^{2n-1-j}\left[\mathbb{1}_{\{j<n\}}\cdot 0+\mathbb{1}_{\{j\geq n\}}\cdot\frac{\binom{n}{j-n}\binom{n}{n}}{\binom{2n}{j}}\cdot 1\right]. \tag{2.7}$$

The utilities in Equations (2.6) and (2.7) should be equal at equilibrium. Solving

[4]Of course the utility depends on the prices of all the other sellers but we suppress this dependence.

the equation we obtain the probability that sellers ask price v.

$$\begin{aligned}
v &= \sum_{j=0}^{2n-1} \binom{2n-1}{j} \left(\rho^A_{(n)}\right)^j \left(1-\rho^A_{(n)}\right)^{2n-1-j} \left[\mathbb{1}_{\{j<n\}} \cdot 0 + \mathbb{1}_{\{j\geq n\}} \cdot \frac{\binom{n}{j-n}\binom{n}{n}}{\binom{2n}{j}} \cdot 1\right] \\
&= \sum_{j=n}^{2n-1} \binom{2n-1}{j} \left(\rho^A_{(n)}\right)^j \left(1-\rho^A_{(n)}\right)^{2n-1-j} \frac{\binom{n}{j-n}}{\binom{2n}{j}} \\
&= \frac{1}{2} \sum_{j=n}^{2n-1} \binom{n-1}{j-n} \left(\rho^A_{(n)}\right)^j \left(1-\rho^A_{(n)}\right)^{2n-1-j} \\
&= \frac{1}{2} \sum_{j=0}^{n-1} \binom{n-1}{j} \left(\rho^A_{(n)}\right)^n \left(\rho^A_{(n)}\right)^j \left(1-\rho^A_{(n)}\right)^{n-1-j} \\
&= \frac{1}{2} \left(\rho^A_{(n)}\right)^n \\
\Rightarrow \rho^A_{(n)} &= (2v)^{1/n}. \qquad (2.8)
\end{aligned}$$

This gives us the following result.

Proposition 1. *In a market where the buyers enter in a random order, the symmetric mixed strategy equilibrium*[5] *is such that the sellers ask price v with probability $\rho^A_{(n)}$ given by*

$$\rho^A_{(n)} = (2v)^{1/n}.$$

Following Satterthwaite and Williams (1989), we measure the market efficiency by

$$M^A_{(n)} = \frac{T_1}{T_0} = \frac{\text{Value created by trades}}{\text{Value created under market clearing}} = \frac{n \cdot 1 + n \cdot \rho^A_{(n)} \cdot v}{n \cdot 1 + n \cdot v} = \frac{1 + (2v)^{1/n} v}{1+v} \qquad (2.9)$$

where T_1 is the total ex-ante value created in equilibrium. T_0 is the total ex-ante value created from trades that would be generated if the market cleared. $M^A_{(n)} = 1$ represents the most efficient case and 0 represents the least efficient. We find that $M^A_{(n)}$ increases in n and approaches 1 as n grows without bound.

$$\frac{\partial M^A_{(n)}}{\partial n} = -\frac{2^{1/n} + v^{1+1/n} \log(2v)}{n^2(1+v)} > 0 \text{ and } \lim_{n\to\infty} M^A_{(n)} = \frac{1 + 1 \cdot v}{1+v} = 1.$$

This shows that in a large economy the inefficiency vanishes. Furthermore, fixing n and

[5] Since we are interested in how random entry and coordination affect market inefficiency, we do not solve the equilibrium explicitly. It is enough to know how many sellers ask price v.

increasing the low-valuation v, $M^A_{(n)}$ decreases for small vs and increases for large vs after reaches its lowest point, say $\widetilde{v}$, i.e, $M^A_{(n)}$ is U-shaped. The derivative

$$\frac{\partial M^A_{(n)}}{\partial v} = \frac{-n + 2^{1/n} v^{1/n} (1 + n + v)}{n(1+v)^2}$$

is negative for small enough v and positive otherwise. We demonstrate the behaviour of $M^A_{(n)}$ numerically in Section 2.4. The interpretation is as follows. As v goes down, the probability $\rho^A_{(n)}$ decreases which indicates that the competition for the high-valuation buyers increases. Consequently, the value created in equilibrium (T_1) increases.

$$\frac{\partial T_1}{\partial v} = n\,(2v)^{1/n} \left(1 + \frac{1}{n}\right) > 0$$

The value of T_1 increases slowly for small vs and then the rate goes up. Meanwhile the value created under market clearing (T_0) also increases in v. However, since T_1 is convex and T_0 is linear, the rates of increase are different, and this leads to the non-monotonicity of $M^A_{(n)}$ as calculated in Equation (2.9).The graphs for T_0 and T_1 are in the appendix.

We have established that the buyers' random entry to the markets associated with the sellers' capacity constraints lead to pricing in mixed strategies. This gives rise to inefficient outcomes where not every profitable trade is consummated. In a small economy, market efficiency, as measured in Equation (2.9), shows that it largely depends on the low-type buyers' valuation for the good, while in a large economy, inefficiency vanishes. Next, we compare these results to a setting where the buyers are able to coordinate their order of entry.

2.3 Coordination

2.3.1 Low-valuations first

In this section we analyze the model where the buyers coordinate on the entry order. To start with, we consider the case where all the low-valuation buyers enter the market first and all the high-valuation buyers enter after them. This could be the first reaction to the inefficiency of the previous section where the low-valuation buyers could not trade as the high-valuation buyers who enter before them grab the low-priced goods. But this turns out to be a bad solution, or the worst possible, as it ignores the sellers' reaction to the entry order. When the low-valuation buyers enter first, the competition for the high-valuation buyers is the highest, and consequently the inefficiencies are also at their highest level.

It is again clear that there is no pure strategy equilibrium, and that in a mixed strategy equilibrium there has to be a mass point at price $p = v$. Denote the probability that the sellers ask price v by $\rho^B_{(n)}$. The utility of asking price unity must be equal to that of asking price v. At the lowest price level v, a seller makes a trade for certain and therefore generates utility v. At the highest price 1, as before, we denote by j the number of the sellers who set price at v. If fewer than half of the sellers choose v, that is, for $j < n$, only j low-valuation buyers trade and the rest, $n - j$, remain unserved. The high-valuation buyers who enter the market after would choose from the cheapest goods available in the market. Therefore the seller who asks the highest price 1 does not trade. If at least half of the sellers choose v, that is, for $j \geq n$, all buyers are served and market clears. Thus the indifference condition is given by

$$\begin{aligned} U(1) &= U(v) \\ \Leftrightarrow \sum_{j=n}^{2n-1} \binom{2n-1}{j} \left(\rho^B_{(n)}\right)^j \left(1 - \rho^B_{(n)}\right)^{2n-1-j} &= v \end{aligned} \tag{2.10}$$

where the probability $\rho^B_{(n)}$ is the solution to the above equation. For any given $n > 0$ and $v \in (0, 1/2)$, the equation has a positive root in the range of $(0, 1)$, a root above 1, a negative root and $2n - 4$ imaginary roots if n is an even number. If n is odd, the equation has one positive root in $(0, 1)$ and $2n - 2$ imaginary roots.

Lemma 3. *For any given $n > 0$ and $v \in (0, 1/2)$, the solution to the probability of asking price v, $\rho^B_{(n)}$, is unique.*

Proof. Rewriting the indifference condition Equation (2.10), we define the function

$$X(\rho^B_{(n)}) = \sum_{j=n}^{2n-1} \binom{2n-1}{j} \left(\rho^B_{(n)}\right)^j \left(1 - \rho^B_{(n)}\right)^{2n-1-j} - v. \tag{2.11}$$

We want to prove that for any $n > 0$ and $v \in (0, 1/2)$, there exists a unique real solution $\rho^B_{(n)} \in (0, 1)$ satisfying $X(\rho^B_{(n)}) = 0$. $X(\rho^B_{(n)})$ is clearly continuous. $X(0) \to -v < 0$

when $\rho^B_{(n)} \to 0$ and $X(1) \to 1 - v > 0$ when $\rho^B_{(n)} \to 1$. By the Intermediate Value Theorem, there exists at least one ρ such that $X(\rho) = 0$. Also by $X'(\rho^B_{(n)}) > 0$ on the interval $(0, 1)$,[6] the function $X(\rho^B_{(n)})$ is monotonically increasing. It therefore shows that for each pair of (n, v), the real number solution to $\rho^B_{(n)}$ in Equation (2.10) is always unique in the range of $(0, 1)$. $\square$

We provide the numerical solution to $\rho^B_{(n)}$ in Section 2.4 which shows that given v, $\rho^B_{(n)}$ monotonically increases in n and approaches 1/2 for large enough ns. Here we first prove the existence of the limit of $\rho^B_{(n)}$ when n grows without bound. We then provide an approximate solution to $\rho^B_{(n)}$ for large ns.

Lemma 4. *As $n \to \infty$, $\rho^B_{(n)}$ approaches 1/2.*

Proof. We view $\{\rho^B_{(n)}\}_{n=1}^{\infty}$ as a sequence of real numbers satisfying $X(\rho^B_{(n)}) = 0$ and denote the limit of the sequence by $\hat{\rho}$ if there exists any. Let $S_{2n-1} = Bin\left(2n-1, \rho^B_{(n)}\right)$ represent a sum of $2n - 1$ Bernoulli-$\rho^B_{(n)}$ random variables X_j. We have

$$\sum_{j=n}^{2n-1} \binom{2n-1}{j} \rho_n^j \left(1 - \rho_n\right)^{2n-1-j} = 1 - Pr\left(S_{2n-1} \leq n - 1\right).$$

Standardising by subtracting the mean and dividing by the standard deviation, the above equation is equal to

$$1 - Pr\left(\frac{S_{2n-1} - (2n-1)\rho^B_{(n)}}{\sqrt{(2n-1)\rho^B_{(n)}\left(1 - \rho_n\right)}} \leq \frac{n - 1 - (2n-1)\rho^B_{(n)}}{\sqrt{(2n-1)\rho^B_{(n)}\left(1 - \rho_n\right)}}\right)$$

Denote the cumulative distribution function of $\frac{S_{2n-1}-(2n-1)\rho^B_{(n)}}{\sqrt{(2n-1)\rho^B_{(n)}(1-\rho_n)}}$ by F_{2n-1}. Then the above expression is equivalent to

$$1 - F_{2n-1}\left(a_n\right) \tag{2.12}$$

where $a_n = \frac{n-1-(2n-1)\rho^B_{(n)}}{\sqrt{(2n-1)\rho^B_{(n)}(1-\rho_n)}}$. The central limit theorem implies that $F_{2n-1}\left(a_n\right) \approx \Phi\left(a_n\right)$ where Φ is the cumulative distribution function of the standardised normal distribution. This holds only for $1 - 2\rho^B_{(n)} \approx 0$, i.e, $\rho^B_{(n)} \approx 1/2$. In Appendix I, we make this rigorous. $\square$

[6]The solution of $X'(\rho^B_{(n)})$ is available upon request.

Lemma 5. *For large ns, the probability of asking price v is approximately given by*

$$\rho^B_{(n)} = \frac{\sqrt{2} + b_{(n)} - \sqrt{2 + \left(b_{(n)}\right)^2}}{2b_{(n)}}$$

where

$$b_{(n)} = \frac{y}{\sqrt{n}}$$

and $\Phi\left(y\right) = 1 - v$ is the distribution function of the standardised normal distribution. For any exogenously given v, the value of y can be found in the Z table.

Proof. We know that for large n, normal distribution is a good approximation for binomial distribution. Let Y_{2n-1} be a binomial random variable with the success probability $\rho^B_{(n)}$. The expression

$$\sum_{j=n}^{2n-1} \binom{2n-1}{j} \left(\rho^B_{(n)}\right)^j \left(1 - \rho^B_{(n)}\right)^{2n-1-j}$$

is thus rewritten as

$$Pr\left(Y_{2n-1} \geq n\right).$$

When n is large, the last expression is about

$$Pr\left(Z \geq \frac{n - \rho^B_{(n)}(2n-1)}{\rho^B_{(n)}\left(1 - \rho^B_{(n)}\right)\sqrt{2n-1}}\right) \approx Pr\left(Z \geq \frac{\sqrt{n}\left(1 - 2\rho^B_{(n)}\right)}{\rho^B_{(n)}\left(1 - \rho^B_{(n)}\right)\sqrt{2}}\right)$$

where Z is the standardised normal distribution. Let

$$b_{(n)} = \frac{1 - 2\rho^B_{(n)}}{\rho^B_{(n)}(1 - \rho^B_{(n)})\sqrt{2}}. \tag{2.13}$$

The probability is simplified as

$$Pr\left(Z \geq \sqrt{n}b_{(n)}\right) = 1 - \Phi\left(\sqrt{n}b_{(n)}\right)$$

where Φ is the distribution function of the standardised normal distribution. Assuming, for instance, that $v = 0.1$, we have

$$1 - \Phi(y) = 0.1 \text{ or } \Phi(y) = 0.9$$

from which we get

$$y \approx 1.28 \text{ or } b_{(n)} \approx \frac{1.28}{\sqrt{n}}.$$

Note that the Z table is required for the value of y. As n goes to infinity,

$$b_{(n)} \to 0 := \hat{b}$$

Rewriting Equation (2.13), we get

$$\rho^B_{(n)} = \frac{\sqrt{2} + b_{(n)} - \sqrt{2 + b^2_{(n)}}}{2b_{(n)}}. \tag{2.14}$$

Since $b_{(n)}$ deceases in n and converges to zero, the approximation of $\rho^B_{(n)}$ also converges. The limit is[7]

$$\rho^B_{(n)} \to 1/2 := \hat{\rho}.$$

□

Proposition 2. *In a market where the buyers coordinate such that all the low-valuation buyers enter the market first, the symmetric mixed strategy equilibrium is such that the*

[7]As $\rho^B_{(n)}$ approaches one half, it may seem that Condition $X(\rho^B_{(n)}) = 0$ cannot hold for large n because v can be anything less than one half. The resolution is that $\rho^B_{(n)}$ grows slowly with n so that the Condition remains valid. We check the rate of convergence.

$$\begin{aligned}
\rho^B_{(n)} &= \frac{\sqrt{2} + b_{(n)} - \sqrt{2 + b^2_{(n)}}}{2b_{(n)}} \\
&= \frac{\sqrt{2} + \frac{1.28}{\sqrt{n}} - \sqrt{2 + (\frac{1.28}{\sqrt{n}})^2}}{2\frac{1.28}{\sqrt{n}}} \\
&\Rightarrow \lim_{n\to\infty} \frac{|\rho_{n+1} - 1/2|}{|\rho_n - 1/2|} = 1 \text{ and } \lim_{n\to\infty} \frac{|\rho_{n+2} - \rho_{n+1}|}{|\rho_{n+1} - \rho_n|} = 1.
\end{aligned}$$

The sequence $\rho^B_{(n)}$ converges to 1/2 sublinearly and logarithmically. The rate of convergence is 1.

sellers ask price v with probability $\rho^B_{(n)}$ given by the indifference condition

$$\sum_{j=n}^{2n-1} \binom{2n-1}{j} \left(\rho^B_{(n)}\right)^j \left(1-\rho^B_{(n)}\right)^{2n-1-j} = v.$$

For large economy, the probability $\rho^B_{(n)}$ approaches $1/2$.

The total ex-ante value created in equilibrium is given by

$$\sum_{j=0}^{n-1} \binom{2n}{j} \left(\rho^B_{(n)}\right)^j \left(1-\rho^B_{(n)}\right)^{2n-j} (j \cdot v + n \cdot 1) + \sum_{j=n}^{2n} \binom{2n}{j} \left(\rho^B_{(n)}\right)^j \left(1-\rho^B_{(n)}\right)^{2n-j} (n \cdot v + n \cdot 1).$$

The first term represents the value created if less than half of the sellers asks price v, that is, at $j < n$. In this case, j low-valuation and n high-valuation buyers would be served and leaves $n - j$ goods unconsumed in the market. The second term represents the value created if at least half of the sellers ask price v, that is, at $j \geq n$. In the later case, all buyers would be served. Thus the market efficiency is measured by

$$\begin{aligned} M^B_{(n)} &= \frac{T_1}{T_0} = \frac{\text{Value created by trades}}{\text{Value created under market clearing}} \\ &= \frac{\sum_{j=0}^{n-1} \binom{2n}{j} \left(\rho^B_{(n)}\right)^j \left(1-\rho^B_{(n)}\right)^{2n-j} (j \cdot v + n \cdot 1) + \sum_{j=n}^{2n} \binom{2n}{j} \left(\rho^B_{(n)}\right)^j \left(1-\rho^B_{(n)}\right)^{2n-j} (n \cdot v + n \cdot 1)}{n \cdot 1 + n \cdot v}. \end{aligned} \tag{2.15}$$

2.3.2 High-valuations first

Next we consider the other polar case where the high-valuation buyers enter the market first and the low-valuation buyers enter afterwards. The high-valuation buyers buy the lowest-priced goods, which leaves some (or all) of the low-valuation buyers unserved depending on how many cheap goods remain by the time they enter the market.

Consider a pure strategy $p = v$ that generates the expected profit v. Clearly there is no profitable deviation. Since high-valuation buyers only choose the cheapest available goods and low-valuation buyers cannot afford the price that is higher than v, a deviation to the price $p' > v$ means there will be no trade and therefore brings zero profit. A deviation to the price $p' > v$ ensures the trade with one of the high-valuation buyers but generates a lower profit. We thus get the pure symmetric equilibrium as follows.

Proposition 3. *In a market where the buyers coordinate such that all the high-valuation buyers enter the market first, the symmetric equilibrium is a pure pricing strategy $p = v$.*

Asking price v brings the utility v to all sellers in the market. It ensures that buyers of both types will be served. The market clears. The efficiency is therefore $M^C = 1$ regardless of the market size, i.e., the value of n.

2.4 Comparison

In this section, we provide numerical solutions and compare the results in the above models. We discuss how market size, valuation of low-type buyers and buyers' entry orders affect market efficiency.

Table 2.1 reports the numerical solutions for the probabilities $\rho^A_{(n)}$, as solved explicitly in Equation (2.8), and $\rho^B_{(n)}$, as shown in Equation (2.10). The market size is represented by $2n$ where the values of n are chosen from 2 to 2000. The valuations of the low-type buyers v are 0.1, 0.2, 0.3 and 0.4, respectively. The solutions to $\rho^B_{(n)}$ is reported in the odd columns and the solutions to $\rho^A_{(n)}$ are provided in the even columns for comparison. As shown in each column, as n grows large, the probability $\rho^A_{(n)}$ in the benchmark

Table 2.1: The probabilities of asking price v in the benchmark and the coordination cases

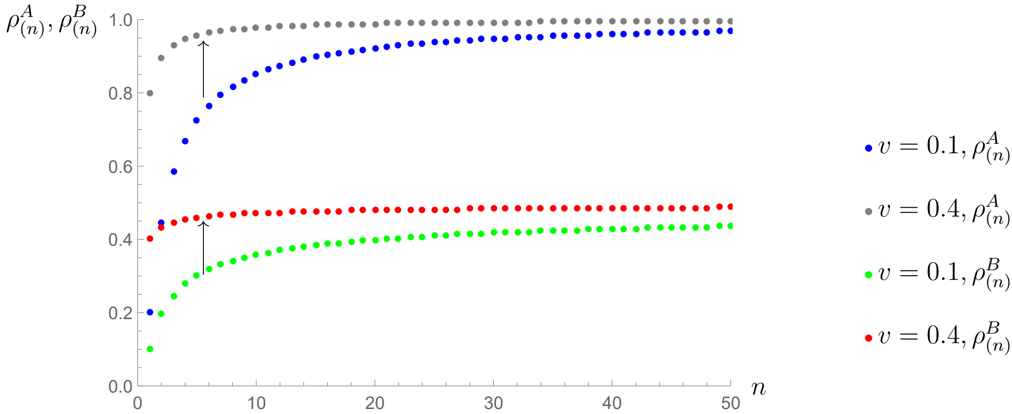
Market size (half) (n) [a]	Valuation (v)							
	0.1		0.2		0.3		0.4	
	$\rho^B_{(n)}$	$\rho^A_{(n)}$	$\rho^B_{(n)}$	$\rho^A_{(n)}$	$\rho^B_{(n)}$	$\rho^A_{(n)}$	$\rho^B_{(n)}$	$\rho^A_{(n)}$
2	0.1958	0.4472	0.2871	0.6325	0.3633	0.7746	0.4329	0.8944
3	0.2466	0.5848	0.3266	0.7368	0.3898	0.8434	0.4463	0.9283
4	0.2786	0.6687	0.3501	0.7953	0.4052	0.8801	0.4539	0.9457
5	0.3010	0.7248	0.3661	0.8326	0.4156	0.9029	0.4590	0.9564
6	0.3177	0.7647	0.3779	0.8584	0.4232	0.9184	0.4627	0.9635
7	0.3309	0.7946	0.3870	0.8773	0.4290	0.9296	0.4656	0.9686
8	0.3415	0.8178	0.3944	0.8918	0.4337	0.9381	0.4679	0.9725
9	0.3504	0.8363	0.4004	0.9032	0.4376	0.9448	0.4698	0.9755
10	0.3579	0.8513	0.4056	0.9124	0.4408	0.9502	0.4713	0.9779
50	0.4360	0.9683	0.4579	0.9818	0.4737	0.9898	0.4873	0.9955
100	0.4547	0.9840	0.4702	0.9909	0.4814	0.9949	0.4910	0.9978
1000	0.4857	0.9984	0.4906	0.9991	0.4941	0.9995	0.4972	0.9998
2000	0.4899	0.9992	0.4933	0.9995	0.4959	0.9997	0.4980	0.9999

[a] The number of sellers and buyers are both $2n$. Thus n in the first column of the table represents half size of the market.

model approaches 1 while the probability $\rho^B_{(n)}$ in the coordination model approaches 1/2, holding v unchanged. As shown in each row, the values of $\rho^A_{(n)}$ and $\rho^B_{(n)}$ increase in v, holding n constant.

Figure 2.1 illustrates how the valuation v affects the probabilities at equilibria, especially in a small economy. The values of $\rho^B_{(n)}$ at $v = 0.1$ (green dots) are much lower

Figure 2.1: The comparison of the probabilities



than at $v = 0.4$ (red dots) for small ns. When the economy grows, dots in both colours move upwards, as the arrow points. The green curve increases quickly and get very close to the red one at $n = 30$. For sufficiently large n, say $n \geq 45$, the values of $\rho^B_{(n)}$ approach 1/2 and become much less dependent on the selection of v. The trend for $\rho^A_{(n)}$ is very similar, as illustrated in blue (for $v = 0.1$) and grey dots (for $v = 0.4$).

We report the measures of market efficiency in Table 2.2. In the odd columns, $M^A_{(n)}$ measures the market performance where the buyers enter the market randomly. In even columns, we report $M^B_{(n)}$ which measures the market performance under the buyers' coordination, as Equation (2.15) shows. The market size is given by $2n$ where n is set to be 5, 10, 50 and 100, respectively. The valuations of low-type buyers are chosen from 0.1 to 0.475. We find that the values of market efficiency in both models have the same trend. In each column, we hold the number n unchanged and let the valuation v vary. We find both $M^A_{(n)}$ and $M^B_{(n)}$ decrease first and then increase. We highlight the cutoff points in blue cells. For each pair of n and v, $M^A_{(n)}$ is always larger than $M^B_{(n)}$. It shows that when the buyers enter the market randomly, market inefficiency is, by comparison, a smaller problem. Buyers' coordinative actions in this way do not help the market clear. In each row, we hold v unchanged and let n increase. We find that the higher

Table 2.2: The measures of the market efficiencies

Valuation (v)	Market size (half) (n)							
	5		10		50		100	
	$M^A_{(n)}$	$M^B_{(n)}$	$M^A_{(n)}$	$M^B_{(n)}$	$M^A_{(n)}$	$M^B_{(n)}$	$M^A_{(n)}$	$M^B_{(n)}$
0.1	0.97498	0.96271	0.98649	0.97330	0.99712	0.98795	0.99855	0.99147
0.125	0.96271	0.95815	0.98562	0.97014	0.99696	0.98655	0.99847	0.99048
0.15	0.97209	0.95465	0.98521	0.96771	0.99690	0.98549	0.99844	0.98973
0.175	0.97179	0.95203	0.98516	0.96590	0.99691	0.98469	0.99845	0.98917
0.2	0.97209	0.95016	0.98541	0.96461	0.99697	0.98412	0.99848	0.98877
0.225	0.97289	0.94891	0.98590	0.96375	0.99709	0.98375	0.99854	0.98850
0.25	0.97411	0.94821	0.98661	0.96327	0.99725	0.98354	0.99862	0.98836
0.275	0.97569	0.94798	0.98748	0.96311	0.99744	0.98347	0.99871	0.98830
0.3	0.97759	0.94816	0.98851	0.96323	0.99765	0.98352	0.99882	0.98834
0.325	0.97975	0.94869	0.98965	0.96359	0.99790	0.98367	0.99895	0.98845
0.35	0.98215	0.94952	0.99092	0.96416	0.99816	0.98392	0.99908	0.98862
0.375	0.98475	0.95062	0.99227	0.96491	0.99844	0.98424	0.99922	0.98885
0.4	0.98753	0.95195	0.99369	0.96582	0.99873	0.98464	0.99936	0.98913
0.425	0.99046	0.95348	0.99519	0.96686	0.99903	0.98509	0.99952	0.98945
0.45	0.99353	0.95518	0.99675	0.96802	0.99935	0.98560	0.99967	0.98981
0.475	0.99671	0.95702	0.99835	0.96929	0.99967	0.98615	0.99983	0.99019

the valuation v is, the higher $M^A_{(n)}$ and $M^B_{(n)}$ would be. It suggests that in a growing economy, the inefficiency vanishes slowly.

Figure 2.2: The comparison of the market efficiencies

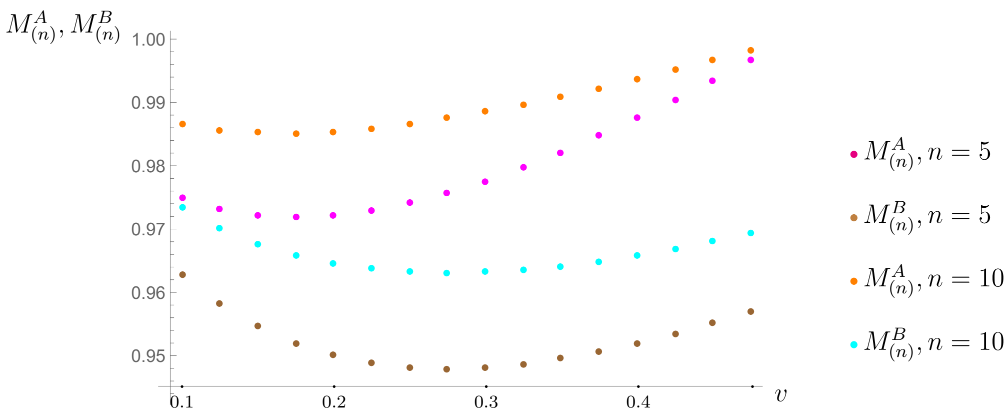


Figure 2.2 illustrates how the market efficiencies $M^A_{(n)}$ and $M^B_{(n)}$ change in v when the values of n are 5 and 10, respectively. They are all U–shaped curves with different lowest points. Given $n = 5$, the value of $M^A_{(n)}$ (magenta dots) drops slightly until it reaches the lowest point 0.971, after which it rises. Meanwhile, the value of $M^B_{(n)}$ (brown dots) is much smaller. It starts from 0.962 (at $v = 0.1$), moves downwards to the lowest point 0.947 (at $v = 0.275$) and increases thereafter. For $n = 10$, $M^A_{(n)}$ and $M^B_{(n)}$ follow a similar trend, as illustrated in orange (for $M^A_{(n)}$) and cyan dots (for $M^B_{(n)}$). The cause of

the trend for $M^A_{(n)}$ has been discussed in the benchmark case and the same argument applies to $M^B_{(n)}$. Recall that market efficiency is measured by the ratio between the total ex-ante value created from trades that each equilibrium generates (T_1) and the total ex-ante value created from trades that would be generated if market clears (T_0). The trend arises because the rate of increase for T_0 in the denominator is different from that of T_1 in the numerator. We provide the graphs for T_1 and T_0 in the appendix.

Note that the measure of market efficiency ($M^C_{(n)}$) is 1 if all the high-valuation buyers enter the market first, as described in Proposition 3. Therefore, such coordination effectively solves the market inefficiency problem.

2.5 Discussion

Our benchmark of buyers entering in a random order is a natural starting point to study the lack of coordination. Then we study the two polar cases in which all the low-valuation buyers entering first might be regarded as a natural solution to the problem where the low-valuation buyers do not get to trade. This, however, ignores the sellers' reactions. Whatever the order of buyers entering, the sellers pricing reflects their competition for the high-valuation buyers. When the low-valuation buyers enter first, fewer trades are consummated and more goods are wasted, which shows that the competition is the fiercest.[8] This manifests in high inefficiency, and shows that competition is not always beneficial. This is no big news but the point is rarely made in a market setting of buyers and sellers. The underlying reason for the inefficiency is that the high-valuation buyers are a fixed and valuable resource, and competition for a fixed resource tends to be wasteful.

In the other polar case the high-valuation buyers enter first. This turns out to be the most efficient order of entry; as the high-valuation buyers are the first in the market there is no need to compete for them. Rather, the pricing, where all the sellers ask price v, reflects the fear of not being able to trade. The situation is very much like the sellers facing a downward sloping demand curve. This order is the choice of a social planner, and it can also be rationalized by the following informal argument.

[8] It is unlike the standard market competition, where fiercer competition means that lower prices attracts more buyers. Here, since the numbers of buyers and goods are fixed, price competition affects the number of trades and therefore the level of market efficiency. The market is fierce in the sense that more goods are wasted and fewer buyers are served.

Instead of entering the markets, assume that the markets are opened at a predetermined time and the buyers arrive in random order to a queue. If they enter the market in this order, the situation corresponds to our benchmark case. Consider the high-valuation buyers in the queue, and allow each of them to ask the person just before them whether the person would like to swap positions for a small sum of money. Each low-valuation buyer would be willing because the best they can expect in the market is buying a good worth v at price v.

If this procedure is allowed to go on for sufficiently many rounds the end result is that all the high-valuation buyers occupy the first half of the position in the queue, and all the low-valuation buyers are at the back. Notice that no high-valuation buyer is willing to pay enough to another high-valuation buyer to swap places. As far as the sellers understand what is going to happen they revise their pricing strategy accordingly.

2.6 Conclusion

Efficiency considerations are of central interest in any market mediated activity, and there are many causes of inefficiency. In this article, we discuss an issue that has not received much attention, that is, the buyers' entry order. We employ a simple game where identical sellers compete for the buyers of heterogeneous valuations. We derive the sellers' equilibrium pricing strategies, and find that in the benchmark model where buyers enter the market randomly, sellers use the mixed strategies in pricing. In equilibrium, not all the low-valuation buyers trade, which leads to market inefficiency. We then compare the equilibrium results in the benchmark with that of two coordination cases where buyers of the same type enter the market at the same time. We find that the sellers react to the buyers' new entry order and use either a pure strategy or a different mixed strategy in pricing. In each of these three cases, the equilibrium is unique.

We then measure the market inefficiencies in these three cases by the ratio between the total ex-ante value created from trades that equilibria generate and the total ex-ante value created from trades that would be generated if market cleared. This comparison provides us with a measure of inefficiency, and also an understanding of how it vanishes as the economy grows. It is notable that in the limit the inefficiency vanishes regardless of the buyers' order of entry. The order only affects the speed of convergence to the efficient outcome.

We study the markets where the goods are not long-lasting so the efficiencies cannot be improved by selling the unsold items at discounted prices. The sellers compete for a fixed amount of buyers with limited resources, unlike the standard competition where lower price and more quantity will attract more buyers. The coordination is designed to relax the competition among the sellers so that more trades are conducted and fewer products are wasted. The possible real-life application of this model can be a fish market or a ticket-selling market for a concert.

Appendix I: Proof of Lemma 4

By Barry-Esseen theorem

$$\left|F_{2n-1}\left(a_n\right)-\Phi\left(a_n\right)\right| \leq \frac{3\,\mathbb{E}\left(|X_1|^3\right)}{\sigma^3\sqrt{2n-1}} = \frac{3\rho^B_{(n)}}{\left(\rho^B_{(n)}\left(1-\rho^B_{(n)}\right)\right)^{3/2}}\frac{1}{\sqrt{2n-1}} \tag{2.16}$$

Since $F_{2n-1}\left(a_n\right)=1-v$ for all n, we get

$$\left|1-v-\Phi\left(a_n\right)\right| \leq \frac{3}{\left(1-\rho^B_{(n)}\right)^{3/2}}\frac{1}{\sqrt{(2n-1)\rho^B_{(n)}}} \tag{2.17}$$

From the indifferent condition (2.10), $v<1/2$ and the above arguments, it is clear that $0<\rho^B_{(n)}<\frac{1}{2}$, and that there is a constant $\alpha>0$ such that $\rho^B_{(n)}\geq\alpha=\frac{1}{4}$.

Since the sequence is closed and bounded, by the Bolzano–Weierstrass Theorem, it is sequentially compact and therefore has a convergent subsequence in $\mathbb{R}^n$. It remains to show that every convergent subsequence of $\left\{\rho_{(n)_k}\right\}_{k=1}^{\infty}$ converges to $\frac{1}{2}$. It is clear that for sufficiently large n, it must be the case that $\frac{1}{8}\leq\rho^B_{(n)}\leq\frac{7}{8}$. Consequently, as n grows indefinitely, the right hand side of Equation (2.17) goes to zero, that is,

$$\Phi\left(a_n\right)\to 1-v\in\left(\frac{1}{2},1\right) \text{ when } n\to\infty.$$

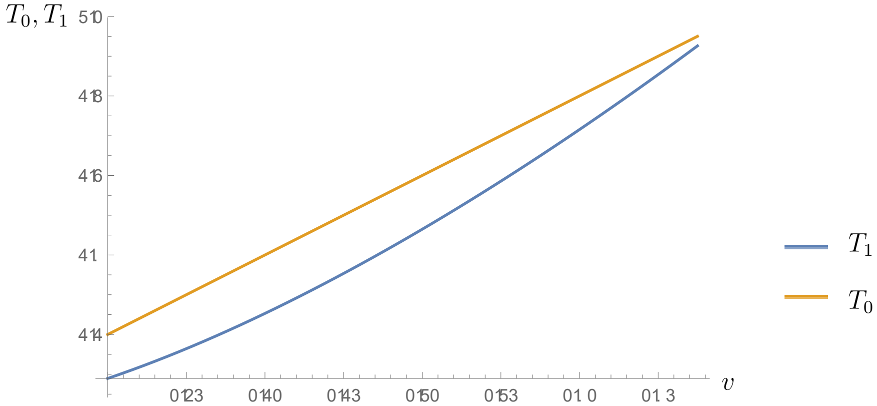
Note that $a_n=\frac{\sqrt{n}\left(1-2\rho^B_{(n)}\right)}{\sqrt{(2-1/n)\rho^B_{(n)}\left(1-\rho^B_{(n)}\right)}}-\frac{1-\rho^B_{(n)}}{\sqrt{(2n-1)\rho^B_{(n)}\left(1-\rho^B_{(n)}\right)}}$ where the first term remains bounded and the second term converges to zero as n grows without limit.

Let us next assume that the subsequence $\{\rho_{n_k}\}_{k=1}^{\infty}\to c$. If $c>\frac{1}{2}$ then $a_{n_k}\to-\infty$, and $\Phi\left(a_{n_k}\right)\to 0$ which is a contradiction. Analogously, if $c<\frac{1}{2}$ then $a_{n_k}\to\infty$, and $\Phi\left(a_{n_k}\right)\to 1$ which is a contradiction.

Since every convergent subsequence converges to $\frac{1}{2}$, by compactness, the whole sequence converges to $\frac{1}{2}$. □

Appendix II: The Values Created in Equilibrium and under Market Clearing

Figure 2.3: The values created in equilibrium and under market clearing



The figure shows how the valuation of low-type buyers (v) affects the total ex-ante value created from trades that equilibrium generates in the benchmark case (T_1) and the total ex-ante value created under market clearing (T_0), when n is set to be 2. For large ns, the two curves are very close to each other, therefore we cannot see their difference properly. T_1 is convex and T_0 is linear. The difference in the rates of increase for T_1 and T_0 is the cause for the trend of market efficiency measures, as stated after Proposition 1. The efficiency $M^A_{(n)}$ evaluated by the ratio between T_1 and T_0 has the trend such that for a given n, the efficiency decreases for small vs, and after some cutoff point, increases. $M^B_{(n)}$ follows a similar trend, as illustrated in Figure 2.2.

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Chapter 3

Socially Responsible Procurement[1]

"There is one and only one social responsibility of business – to ... engage in activities designed to increase its profits."

Milton Friedman (1970)[2]

3.1 Introduction

Authorities all over the world have established guidelines for socially responsible procurement. The EU has recently published voluntary criteria for green public procurement (for public space maintenance, for food, catering services and vending machines, etc. in 2019). The US has several programmes that promote environmentally-friendly procurement (environmentally preferable purchasing, comprehensive procurement guidelines, green procurement programmes, etc.).[3] Also, the OECD has launched a program to advance the consideration of responsible business practices in public procurement, encompassing various environmental, social and economic aspects (including, e.g., human rights and labour rights). These policies are designed to change the incentives of firms. Indeed, empirical research shows that firms who invest more in corporate social responsibility (CSR) receive more contracts in procurement (Flammer, 2018) and receive a higher return on their lobbying spending (Garcia, 2016). Why exactly this happens is not yet very well understood.

[1]This chapter is based on an article jointly written with Saara Hämäläinen.

[2]Friedman M (1970). The social responsibility of business is to increase its profits. *The New York Times Magazine*, 13 September, 1970, available at: `http://umich.edu/~thecore/doc/Friedman.pdf` (accessed on 12 January, 2020).

[3]See `https://ec.europa.eu/environment/gpp/index_en.htm` and `https://www.fedcenter.gov/programs/buygreen/index.cfm?` (accessed on 13 January, 2020).

To provide a framework for analysis, this paper studies socially responsible procurement in a model of a lobbying contest. Firms can try to increase their competitiveness in the market for government contracts both directly by lobbying and by investing in CSR, which is regarded favourably by the tendering authority. We study how socially responsible procurement affects the money that firms allocate to influence the outcome of the contest. An important assumption is that comparing firms' relative merits becomes harder for the authority if the public relations (PR) strategy of a firm relies partly on investment in CSR. There are several reasons to rationalise this assumption. First, there are many guidelines that encourage the authority to select a more socially responsible firm. There is no universally accepted guideline to define socially responsible actions and rank their importance. Second, the comparison of firms' CSR investment requires clear rules to assess their impacts or to quantify the quality of the investment, for example, to what extent a project can contribute to society. However, there is no clear rule for this purpose. The comparison depends greatly on the authority's personal judgement. Third, CSR investment can take various different forms. It is highly unlikely that the two competing firms invest in identical projects and the spending on each project is also the same. A firm's socially responsible profile usually consists of a set of actions. These jointly make it difficult for the authority to compare two firms' merits and drawbacks and consequently make it difficulty to determine the more suitable winner in the lobbying contest. Fourth, misinformation from dishonest firms is a growing issue. Firms that claim to be environmentally friendly may not be fully trustworthy.[4]

There exists, for example, the MSCI KLD scoring system[5] that is widely used by investors to evaluate firms' ethical and socially responsible performances. KLD Research & Analytics Inc. gives scores to firms that invests in or make commitments on the following five aspects: (i) environment, (ii) community and society, (iii) employees and supply chain, (iv) customers, and (v) governance and ethics. The scores help conscious investors choose more ethical and socially responsible firms, but are typically not applicable for the authority reviewing the proposals of firms competing for a public contract, leaving the authority with the task of comparing the firms in terms of multiple,

[4] See for example, `https://www.theguardian.com/business/2019/mar/22/top-oil-firms-spending-millions-lobbying-to-block-climate-change-policies-says-report`

[5] MSCI KLD stands for Morgan Stanley Capital International, Kinder, Lydenberg, Domini & Co.

diverse environmental, social and economic measures.

In this paper, we show that socially responsible procurement, thus, (i) alleviates the competition among firms for government contracts, (ii) shifts firms' PR spending from lobbying to CSR investment, and (iii) decreases the total amount of money that firms use to influence the authority. This is welfare improving because expenditures on lobbying are typically at least partially socially wasteful. As a result, we find that socially responsible procurement is actually *socially responsible* in the sense of reducing inefficient spending. Firms are also better off because they no longer need to compete their profits away in lobbying. Investing in CSR is hence shown to be rational even for firms that care only about their profits. In general, our finding of *incomplete rent dissipation* in contests where firms can differentiate their political strategies may also help to reconcile the Tullock (2001) paradox of why firms spend so little in lobbying for often substantial rents.[6]

The basic model is deliberately simple. There are two firms competing for a contract which gives the winner some profit. The decision of which contractor to choose is delegated to a government authority. To influence the authority, firms can spend money on PR, with the options of either spending it solely on lobbying or partly investing in CSR. The total spending otherwise affects the outcome as in a standard lobbying game: the firm with higher spending wins the prize. However, with the introduction of CSR investments, the authority finds it difficult to compare the value of the investment and thus to determine a more suitable winner. The difficulty comes from the fact that the value and effect of the investment is under the authority's personal judgement. The authority may not view an investment as socially responsible or may face difficulties in assessing and quantifying the effectiveness of a CSR investment. Also, it is highly unlikely that two firms would invest in the same socially responsible projects and the spending is also exactly identical so that the investment can be directly compared.[7]

We first characterize the equilibrium for the case of symmetric firms. The equilibrium turns to be in mixed strategies. This entails that firms mix in spending on lobbying and investing in CSR. One interpretation is that in some years a firm's PR budget

[6] In their influential paper, Ansolabehere et al. (2003) also ask why is there so little money in the US political lobbying, and recently Borisov et al. (2015) show that lobbying increases the stock market value of a firm, which also suggests that rents are not fully dissipated in the process of lobbying.

[7] The model is quite similar to that of Carlin (2009), employed to analyze consumer obfuscation in financial products markets, but here his price competition model is replaced by an all-pay auction, which is the standard way of analyzing procurement contests in the literature.

features only lobbying while in other years the firm allocates most of this budget to CSR investment. As a typical finding in all-pay auctions (Baye et al., 1996), each firm also economizes on cost by randomizing the total amount spent on influencing the contest outcome. Interestingly, we find that lobbying is a more competitive strategy and CSR spending a less competitive strategy for a firm. The decision between the two strategies depends on the firm's (random) level of expenditure: The firms whose expenditures fall below a cutoff level prefer to spend all their money on CSR (to alleviate competition) and firms who spend more prefer to spend their money on lobbying (to intensify competition). This is because lobbying can lead to head-on competition where the firm that spends more always wins. If the firm invests in CSR instead, the authority is compelled to take it into consideration, although it is difficult to estimate – giving the firm a chance to win on a low budget.[8] In an extension, we analyze the case where firms differ. Then we find that the more efficient firm, which gains more from winning the contest, is more competitive and is thereby more likely to lobby and not spend on CSR. We discuss possible interpretations and exceptions to this rule in the paper.

There are different approaches to CSR in economics. On the one hand, Bénabou and Tirole (2010) suggest that spending on CSR can sometimes be regarded as a firm's adoption of a more long term perspective. Indeed, Kempf and Osthoff (2007) show that socially responsible investing where companies are selected based on CSR leads to high abnormal returns. On the other hand, Kotchen and Moon (2012) find empirical support for the hypothesis that companies invest in CSR to offset corporate social *irresponsibility*. In our paper, CSR is instead an welfare improving alternative to a more socially costly behavior, i.e., lobbying. Besley and Ghatak (2007) argue that firms do not have a comparative advantage in CSR investment even when consumers desire it because a free-riding problem arises in private provision. Our paper differs from earlier work in that we do not focus on the desirability of CSR but on its strategic use by firms in socially responsive procurement where the authority regards various investments in CSR favourably; Baron (2001) considers strategic use of CSR by firms in a more general set-up where rival firms are targeted by activists.

Our analysis connects (i) the industrial organization literature on differentiation and

[8] Even when the authority is so corrupt that it only cares about expenditures that benefit it directly, established guidelines for sustainable procurement likely compel it to consider investment in CSR as well.

obfuscation and (ii) the public choice literature that analyzes rent-seeking in lobbying contests. For excellent surveys of contest literature, see Nitzan (1994) and Corchón et al. (2018). Investing in CSR is an efficient lobbying strategy in our model because it enables a firm to differentiate from its rival and make the choice among firms harder for the authority.[9] This is shown to relax the competition for government contracts, similarly to what happens in markets (see Perloff and Salop, 1985; Wolinsky, 1986; Shaked and Sutton, 1982). There is also a link to the literature on strategic complexity and obfuscation in consumer markets, e.g., Gabaix and Laibson (2006); Ellison and Wolitzky (2012); Wilson (2010); Piccione and Spiegler (2012); Chioveanu and Zhou (2013), which observe that firms have incentives to make their products harder for consumers to analyze. This has adverse effects on consumers and market welfare.[10] However, our paper shows that in contests the effects of strategic complexity could instead be positive since, by alleviating competition, it additionally reduces wasteful spending. This is because competition is not usually productive in lobbying, unlike in markets.[11]

Explanations of the Tullock's paradox (Tullock, 2001) have previously concentrated on, e.g., asymmetric payoffs (Hillman and Riley, 1989) and war of attrition type competition (Riley, 1999). We observe that spending is more limited with socially responsible procurement where firms can affect the government authority by investing in CSR instead of lobbying. Technically, we find that by investing in CSR a firm can introduce a "consolation prize", to be allocated to the firm that loses the game, lowering the incentive to win the "first prize". In the literature, the availability of a consolation prize and noise in a contest is known to limit lobbying spending on contests with multiple prizes (Barut and Kovenock, 1998) and on contests with additive noise (Long, 2013). However, the trade-off between the first prize and the consolation price that firms encounter here is a novelty. The question is related to optimal contest design (Kirkegaard, 2012; Siegel, 2014; Seel and Wasser, 2014; Haan, 2016). The twist in our paper is that contest payoffs derive from the strategies of the contestants and not from that of a contest designer.

[9] For a classic article on environmental product differentiation, see Reinhardt (1998).

[10] But see Taylor (2017), where obfuscation allows screening and improves welfare.

[11] As known since Tullock (1967), the possibility of acquiring a monopoly position by influencing public choice not only reduces welfare by alleviating competition in product markets (captured by the "Harberger triangle" of dead weight loss) but also generates losses because of unproductive lobbying competition (represented by the "Tullock square" of dissipated rents).

The paper is organized as follows. The model is first set up in Section 3.2 and the nature of equilibrium strategies described in Section 3.3. Then, we characterize the equilibrium in symmetric procurement contests with CSR investment in Section 3.4 and the case of asymmetric contests in Section 3.5. Section 3.6 discusses various extensions of our analysis. Section 3.7 concludes the analysis.

3.2 Model

Two firms $i = 1, 2$ compete for a contract, which gives the winning firm i a profit of T_i. The decision of choosing the contractor is delegated to a government authority. The authority works under guidelines that require it to favour socially responsible business practices.

To influence the authority, firms allocate money to PR. Two firms move simultaneously. The action set is (e_i, c_i) which represents the total spending on PR $e_i \in [0, T_i]$ and the share allocated to CSR investment $c_i \in \{L, H\}$, respectively. The action H represents a high proportion of the total expenditure on socially responsible projects and the complementary action L represents a low proportion.[12] For consistent language and simplification purposes, from now on, **efforts** (e) refer to the total spending and **investments** (c) refer to the strategic spending on CSR related matters for the purpose of differentiating itself from the competitor. Firms decide their effort and investment levels at the same time.

When no firm invests in CSR, the authority compares the efforts directly like in a standard all pay auction: the firm that exerts higher effort wins. When one or both invest, the value and effect of the investment is under the authority's personal judgement. The authority's difficulty to assess and evaluate the value of the investment is measured by $\delta(c_i, c_j)$. With probability $1 - \delta(c_i, c_j)$ where $i \neq j$ and $i, j \in \{1, 2\}$, the authority favours the investment and is able to assess the quality of the investment properly. That is, the value of both firms' investment can be quantified and thus compared directly. However, with probability $\delta(c_i, c_j)$, the authority is uncertain about the quality of the investment, i,e, whether the invested projects are socially responsible

[12]For simplicity, we let the CSR investment level be binary. Alternatively, we could let c_i vary in the range of $[0, 1]$ and the corresponding probability that the difficulty of determination occurs, $\delta(c_i, c_j)$, increases in both arguments c_i and c_j where $i \neq j$, that is $\frac{\partial \delta}{\partial c_i} > 0$. Such settings complexify the calculation but do not change the main results.

and to what extent they contribute to society. Consequently, it causes the difficulty to determine the more suitable winner among two competing firms. Furthermore, the high investment level ($c_i = H$) is assumed to consist of a more complex portfolio of projects related to CSR. The low investment level ($c_i = L$) is assumed to be more simple and transparent so it requires much less and easier assessment. That is, $\frac{\partial \delta}{\partial H} > \frac{\partial \delta}{\partial L} > 0$. From now on, **uncertainty** (δ) refers to the probability that the difficulty of determination occurs.

Table 5.2 shows the firms' choices in the investment and the authority's uncertainty levels that are caused by these choices. When both firms choose the high investment

Table 3.1: Firms' CSR investment selection and the corresponding uncertainty level of the authority

		Firm j: L	Firm j: H	
Firm i	L	$\delta_0 = 0$	δ_1	λ
	H	δ_1	δ_2	$1-\lambda$
		λ	$1-\lambda$	

Firms i and j choose the investment level $c_i, c_j \in \{L, H\}$ where L represents the low and H represents the high level, $i \neq j$ and $i, j \in \{1, 2\}$. The probability of choosing L is $\lambda \in [0, 1]$. For each pair of investment levels (c_i, c_j), it results in a certain level of uncertainty in determination and this happens with probability $\delta(c_i, c_j) \in [0, 1]$, where $0 =: \delta_0(L, L) < \delta_1(H, L) < \delta_2(H, H) \leq 1$.

level H, the uncertainty reaches its highest at $\delta_2(H, H) \leq 1$. When both choose L, without loss of generality, we normalise the uncertainty $\delta_0(L, L)$ to be zero. That is, the winning prize rewards the firm with the higher effort. Naturally, if one and only one firm chooses H, the uncertainty $\delta_1(H, L) = \delta_1(L, H)$ is medium. The profit of a firm is thus given by

$$\Pi_i(e_i, c_i, c_j) = T_i \Big\{ \underbrace{\delta(c_i, c_j)\frac{1}{2}}_{\text{difficulty in comparison}} + \overbrace{\big[1 - \delta(c_i, c_j)\big] F_j(e_i)}^{\text{no diffculty}} \Big\} - e_i, \tag{3.1}$$

where for the mixed strategy over e_i with continuous support, $F_j(e_i)$ denotes firm j's cumulative distribution of effort. With probability $\delta(c_i, c_j)$, the authority determines the winner by a draw, that is, each firm has an equal chance of winning. With probability

$1-\delta(c_i, c_j)$, the authority determines the winner according to the effort directly.

Firm i's expected profits by choosing the investment $c_i \in \{L, H\}$ and the effort e_i are

$$\Pi_i(L, e_i) = \lambda[TF_L(e_i)] + (1-\lambda)[T\delta_1/2 + T(1-\delta_1)F_H(e_i)] - e_i$$
$$\Pi_i(H, e_i) = \lambda[T\delta_1/2 + T(1-\delta_1)F_L(e_i)] + (1-\lambda)[T\delta_2/2 + T(1-\delta_2)F_H(e_i)] - e_i \tag{3.2}$$

where $F_{c_j}(e_i)$ is the cumulative distribution conditional on the choice on firm j's investment level $c_j \in \{H, L\}$. $\lambda \in [0, 1]$ is the probability of choosing the low investment level L.

3.3 Mixed Strategies

The first observation we make in Lemmas 6 and 7 is that firm use mixed strategies in effort and investment.

Lemma 6. *Firms use mixed strategies in effort* (e_i).

Proof. The proof of the lemma is straightforward and similar as, for example, Baye et al. (1996) which shows that both firms use a continuous mixed bidding strategy in all pay auctions. Consider a candidate equilibrium with pure strategies in effort (e_i, e_j) and (pure or mixed) strategies in investment (c_i, c_j). First, if $e_i < e_j$, there is a profitable deviation for firm j to a slightly lower effort level $e' \in (e_i, e_j)$, which reduces its costs without affecting its winning probability. Second, if $e_i = e_j$, there is a profitable deviation for firm i to a marginally higher effort level $e' > e_j$, increasing its probability of winning from 1/2 to $\delta(c_i, c_j)\frac{1}{2} + 1 - \delta(c_i, c_j)$, with only a slight positive effect on its cost. This shows that there is no equilibrium where firms apply pure strategies in effort. □

Lemma 7. *Firms use mixed strategies in investment* (c_i).

Proof. We prove by contradiction. To begin, suppose that firms apply pure strategies $(c_i, c_j) = (L, L)$. The game becomes a standard all-pay auction. Thus, $e_i = 0$ is within the support of a firm's optimal strategy in the tentative equilibrium. The choice of $(e_i, c_i) = (\epsilon, L)$ yields the firm the payoff of $F_j(\epsilon)T_i - \epsilon \to 0$, as $\epsilon \to 0$ and $F_j(\epsilon)$ represents the probability that firm i wins as effort ϵ goes to 0. However, by choosing instead $(e_i, c_i) = (\epsilon, H)$ (zero effort together with higher investment), the firm obtains a payoff of $T_i(\delta(H,L)/2 + (1-\delta)F_j(\epsilon)) - \epsilon \to T_i\delta(H,L)/2$, as $\epsilon \to 0$ and $F_j(\epsilon) \to 0$, which is strictly larger. This constitutes a profitable deviation. [13]

Next, suppose that firms apply pure strategies $(c_i, c_j) = (H, H)$. Firm i's expected profit is given by

$$\Pi_i(c_i, c_j, e_i) = \Pi_i(H, H, e_i) = T_i[\delta_2 \frac{1}{2} + (1-\delta_2)F_H(e_i)] - e_i$$

where $F_H(e_i)$ is the cumulative distribution when firm j chooses $c_j = H$ and firm i's effort e_i is continuous in the support. If $\delta_2 = 1$, firm i obtains the maximum profit $\frac{T}{2}$ at $e_i = 0$. A deviation to $(c_i', c_j) = (L, H)$ results in the change in the difficulty level from $\delta_2(H,H)$ to $\delta_1(L,H)$. Consider a small positive effort level $e_i' > e_i = 0$ such that $F_H(e_i') > 0$. To be a profitable deviation, the following inequality has to be satisfied.

$$\begin{aligned} \Pi_i'(c_i', c_j, e_i') = \Pi_i'(L, H, e_i') = T_i[\delta_1 \frac{1}{2} + (1-\delta_1)] - e_i' > T_i \frac{1}{2} = \Pi_i(c_i, c_j, e_i) \\ \Rightarrow e_i' < \frac{T}{2}(1-\delta_1) \end{aligned}$$

For any δ_1, there exists some $e_i' > 0$ such that the last inequality is satisfied. Therefore the candidate pure strategy $(c_i, c_j) = (H, H)$ cannot be an equilibrium.

If $\delta_2 < 1$, suppose the candidate equilibrium effort is e_i. As with the above arguments, we consider a slightly higher effort $e_i' > e_i = 0$ and $c_i' = L$ that generates the profit

$$\Pi_i'(c_i', c_j, e_i') = \Pi_i'(L, H, e_i') = T_i[\delta_1 \frac{1}{2} + (1-\delta_1)F_H(e_i')] - e_i' = T_i[1 - \delta_1 \frac{1}{2}] - e_i'$$

$$\Pi_i'(c_i', c_j, e_i') > \Pi_i(c_i, c_j, e_i)$$

iff

$$T_i[1 - \delta_1 \frac{1}{2}] - e_i' > T_i[\delta_2 \frac{1}{2} + (1-\delta_2)F_H(e_i)] - e_i$$

[13] The proof holds for both symmetric and asymmetric case. For $T_1 \neq T_2$, firms use mixed strategy in effort $e_i \in [0, T_{min}]$, where $T_{min} := \min\{T_1, T_2\}$.

that is

$$T_i[\frac{1}{2}(\delta_1 + \delta_2) - 1 - (1 - \delta_2)F_H(e_i)] < e'_i - e_i < 0$$
$$\Rightarrow \frac{1}{2}(\delta_1 + \delta_2) - 1 - (1 - \delta_2)F_H(e_i)] < 0$$

Since $F_H(e_i)$ in monotonically increasing in e_i, we only need to check whether the last expression holds at the boundary. When $F_H(e_i) = 0$, we get $\delta_1 + \delta_2 < 2$. When $F_H(e_i) = 1$, we get $\delta_1 + 3\delta_2 < 4$. That is to say, for any $F_H(e_i) \in [0, 1]$, there exists a slightly higher effort level $e'_i > e_i$, such that $F_H(e'_i) = 1$ and $\Pi'_i(e'_i) > \Pi_i(e_i)$.

Last, suppose that firms apply pure strategies where only one firm chooses high investment, that is, $(c_i, c_j) = (H, L)$. Suppose the candidate equilibrium effort is e_i that generates

$$\Pi_i(c_i, c_j, e_i) = \Pi_i(H, L, e_i) = T_i[\delta_1\frac{1}{2} + (1 - \delta_1)F_L(e_i)] - e_i.$$

Consider a slightly higher effort $e'_i > e_i$ and $c'_i = L$ such that

$$\Pi'_i(c'_i, c_j, e'_i) = \Pi_i(L, L, e'_i) = T_i[\delta_0\frac{1}{2} + (1 - \delta_0)F_L(e'_i)] - e'_i = T_i - e'_i.$$

$$\Pi'_i(c'_i, e'_i) > \Pi_i(c_i, e_i)$$

iff

$$T_i - e'_i > T_i[\delta_1\frac{1}{2} + (1 - \delta_1)F_L(e_i)] - e_i,$$

that is

$$\frac{1}{2}\delta_1 + (1 - \delta_1)F_L(e_i) - 1 < 0.$$

For the same reason as above, we check the boundary. Since the last equality is satisfied when $F_L(e_i) = 0$ and $F_L(e_i) = 1$, it completes the proof. There is no pure investing strategy equilibrium.[14] □

The intuition for Lemma 7 is as follows. Firms use mixed strategies in CSR investment because choosing $c_i = L$ allows a firm to intensify competition (reduce δ) when its PR spending is relatively higher (closer to the upper bound of equilibrium expenditure $\bar{e}$); whereas choosing $c_i = H$ enables a firm to alleviate competition (elevate

[14]There is a pure equilibrium in CSR investment if and only if $\delta_1 = \delta_2$. The equilibrium strategies are $(c_1, c_2) = (H, H)$.

δ) when its PR spending is relatively lower (more close to the lower bound of equilibrium expenditure $\underline{e}$). Both options are thus valuable for a firm but in different situations. We find that, since firms use mixed strategies in PR expenditures e_i over the same support $[\underline{e}, \overline{e}]$ as Lamma 6 proves, they must use mixed strategies in their CSR investments c_i as well.

In the case where no CSR investment is made, i.e., both firms choose $c_i = L$ for $i = 1, 2$, the contest reduces to an all-pay auction where at least one firm makes zero expected profit. It competes away its profits in the contest.[15] In the case where at least one firm invests in CSR, firms may reduce the spending on PR and benefit from the authority's difficulty in choosing the winner. Both firm expect positive profits. Thus, the incentive to alleviate competition by investing in CSR with a positive probability arises.

Further, note that our assumptions on payoffs also entail that the *contest success function* in Eq. (3.1) corresponds to that in a contest with a winning prize of size $W_i = T_i(1 - \delta/2)$ and a consolation prize of size $C_i = T_i\delta/2$, which is positive if δ is positive. The winning prize is given by $T_i(1 - \delta) = W_i - C_i = W - T_i\delta/2$, resulting in the following payoff function

$$\Pi_i = C_i + (W_i - C_i)\, F_j(e_i) - e_i.$$

Technically, this explains why investing in CSR, which gives rise to a positive δ, alleviates competition in contests. In the contest literature, e.g., Barut and Kovenock (1998), the availability of a consolation prize is known to limit competition. Here, CSR investment by firms introduces a consolation prize, which benefits firms as it guarantees them higher profits.

3.4 Symmetric contests

Next, we concentrate on a symmetric game where payoffs are the same, $T_1 = T_2 =: T$, and firms have the same incentives in competing for a contract.

Lemma 8 (Symmetric Contest)**.** *In equilibrium, there is a level $e_i^\star$ such that a firm chooses low CSR investment, i.e., $c_i = L$, with higher effort, for $e_i > e_i^\star$, and high CSR*

[15] The arguments are the same as in, for example, Ellingsen (1991), p. 650.

investment, i.e., $c_i = H$*, with lower effort, for* $e_i < e_i^\star$*.*

Proof. We denote the range associated with the low investment at the equilibrium as $S_L = [\underline{e_L}, \overline{e_L}]$ and the range associated with high as $S_H = [\underline{e_H}, \overline{e_H}]$. We prove in four steps.

1. $S_L \cap S_H \neq \emptyset$

   We prove by contradiction. a) Suppose, at equilibrium, $\underline{e_L} > \overline{e_H} > 0$ so that $S_L \cap S_H = \emptyset$, from Equation (3.2), we get $F_H(e_L) = 1, \forall e_L \in S_L$. By choosing $c_i = L$ and $e_i = \underline{e_L}$, firm i obtains the profit $\Pi_i(L, \underline{e_L}) = (1-\lambda)T_i(1-\delta_1/2) - \underline{e_L}$ by $F_L(\underline{e_L}) = 0$. The profit is positive only if firm j chooses the investment H. Firm i can do better by deviating to $e' = \underline{e_L} - \epsilon > \overline{e_H}$ where $\epsilon > 0$.

   b) Suppose $\underline{e_H} > \overline{e_L} > 0$, we get $F_H(e_L) = 0, \forall e_L \in S_L$. By choosing $c_i = L$ and $e_i = \underline{e_L}$, firm i obtains the profit $\Pi_i(L, \underline{e_L}) = (1-\lambda)T_i\delta_1/2 - \underline{e_L}$. The firm can do better by choosing H instead and thus gets $\Pi(H, \underline{e_L}) = \lambda T_i\delta_1/2 + (1-\lambda)T\delta_2/2 - \underline{e_L} > \Pi(L, \underline{e_L})$.

2. $\min\{\underline{e_L}, \underline{e_H}\} = 0$

   Suppose not. $\min\{\underline{e_L}, \underline{e_H}\} = \underline{e_L} = e > 0$, for any $F(e) = 0$, there would be a profitable deviation $e' < e$ s.t. $F(e') = 0$ and $U(e) < U(e') < 0$ for positive e' and $U(e') = 0$ for $e' = 0$.

3. $S_L \cap S_H = \{e\}$ for some e

   Suppose $S_L \cap S_H = [\underline{e}, \overline{e}]$, then for any $e \in [\underline{e}, \overline{e}]$, the indifference condition $\Pi(L, e) = \Pi(H, e)$ should hold.

$$\begin{aligned}
&\lambda[T_iF_L(e)] + (1-\lambda)[T_i\delta_1/2 + T_i(1-\delta_1)F_H(e)] - e \\
= {}&\lambda[T_i\delta_1/2 + T_i(1-\delta_1)F_L(e)] + (1-\lambda)[T_i\delta_2/2 + T_i(1-\delta_2)F_H(e)] - e \\
&F_L(e)\{\lambda\delta_1\} + C = F_H(e)\{(1-\delta_2)(\delta_1-\delta_2)\}.
\end{aligned}$$

   where $C = \frac{1}{2}(\delta_1 - 2\lambda\delta_1 - (1-\lambda)\delta_2)$ does not depend on e. Since $F_L(e)$ and $F_H(e)$ increase in e, LHS is increasing in e while RHS is decreasing. Contradiction.

4. $S_H = [\underline{e_H}, \hat{e}]$ and $S_L = [\hat{e}, \overline{e_L}]$

Suppose not. $S_L = [\underline{e_L}, \hat{e}]$ and $S_H = [\hat{e}, \overline{e_H}]$. Clearly $\underline{e_L}$ must be zero. A firm's profit from choosing low investment L and low effort $\underline{e_L}$ is $\Pi(L, \underline{e_L}) = (1-\lambda)T(1-\delta_1/2) - \underline{e_L}$ by $F_L(\underline{e_L}) = F_H(\underline{e_L}) = 0$. It only obtains positive profit if the other firm chooses high investment H. Now consider a deviation to $(H, \underline{e_L})$. $\Pi(H, \underline{e_L}) = \lambda T_i \delta_1/2 + (1-\lambda)T\delta_2/2 - \underline{e_L} > \Pi(L, \underline{e_L})$. It is a profitable deviation. Contradiction.

Therefore we show that at equilibrium, $S_H = [\underline{e_H}, \hat{e}] = [0, \hat{e}]$ and $S_L = [\hat{e}, \overline{e_L}]$. Note that the arguments hold for $T_1 \neq T_2$ and we will use this result in Section 3.5 to solve the asymmetric case. $\square$

Now we derive the mixed strategy equilibrium. The probability of choosing low investment L, λ, is given by the indifference condition $\Pi(L, \hat{e}) = \Pi(H, \hat{e})$. By $F_L(\hat{e}) = 0$ and $F_H(\hat{e}) = 1$, we get

$$\begin{aligned}
\lambda[TF_L(\hat{e})] + (1-\lambda)[T\delta_1/2 + T(1-\delta_1)F_H(\hat{e})] - \hat{e} = \\
\lambda[T\delta_1/2 + T(1-\delta_1)F_L(\hat{e})] + (1-\lambda)[T\delta_2/2 + T(1-\delta_2)F_H(\hat{e})] - \hat{e} \\
\Rightarrow \lambda = (\delta_2 - \delta_1)/\delta_2 \text{ and } 1-\lambda = \delta_1/\delta_2
\end{aligned} \tag{3.3}$$

Plug λ in $\Pi(H, \underline{e_H})$, we solve the equilibrium payoff

$$\begin{aligned}
\Pi(H, \underline{e_H}) = \Pi(H, 0) &= \lambda T\delta_1/2 + (1-\lambda)T\delta_2/2 - \underline{e_H} \\
&= T\delta_1(\delta_2 - \delta_1)/(2\delta_2) + T\delta_1/2 > 0
\end{aligned}$$

We compare the payoff to that of Ellingsen (1991), which generates zero expected profit, and find that a firm obtains a higher profit by investing in CSR related matters. The upper bound of S_H, $\hat{e}$, is given by $\Pi(H, \hat{e}) = \Pi(H, 0)$.

$$\begin{aligned}
\lambda T\delta_1/2 + (1-\lambda)T(1-\delta_2/2) - \hat{e} = \lambda T\delta_1/2 + (1-\lambda)T\delta_2/2 - \underline{e_H} \\
\Rightarrow \hat{e} = (1-\lambda)T(1-\delta_2) = \delta_1/\delta_2 T(1-\delta_2).
\end{aligned} \tag{3.4}$$

The upper bound of S_L, $\overline{e_L}$, is given by $\Pi(L, \overline{e_L}) = \Pi(L, \hat{e}) = \Pi(H, \hat{e}) = \Pi(H, 0)$.

$$\begin{aligned}
&\lambda T + (1-\lambda)T(1-\delta_2/2) - \overline{e_L} \\
&= \lambda T\delta_1/2 + (1-\lambda)T\delta_2/2 - \underline{e_H} \\
&\Rightarrow \overline{e_L} = T(1-\delta_1).
\end{aligned} \tag{3.5}$$

The effort distribution associated with high investment, $F_H(e)$, is given by $\Pi(H, e) = \Pi(H, \underline{e_H})$ and $F_L(e) = 0$.

$$\begin{aligned}
&\lambda T\delta_1/2 + (1-\lambda)\{T\delta_2/2 + T(1-\delta_2)F_H(e)\} - e \\
&= \lambda T\delta_1/2 + (1-\lambda)T\delta_2/2 - \underline{e_H} \\
&\Rightarrow F_H(e) = \frac{e}{(1-\lambda)T(1-\delta_2)} = \frac{e\delta_2}{\delta_1 T(1-\delta_2)}.
\end{aligned} \tag{3.6}$$

The effort distribution associated with low investment, $F_L(e)$, is given by $\Pi(L, e) = \Pi(H, \underline{e_H})$ and $F_H(e) = 1$.

$$\begin{aligned}
&\lambda[TF_L(e)] + (1-\lambda)[T\delta_1/2 + T(1-\delta_1)] - e \\
&= \lambda T\delta_1/2 + (1-\lambda)T\delta_2/2 - \underline{e_H} \\
&\Rightarrow F_L(e) = \frac{e - T + \lambda T + \delta_1 T - \lambda T\delta_1/2}{\lambda T} \\
&= \frac{\delta_1 T - \delta_2(e + \delta_1 T)}{(\delta_1 - \delta_2)T}.
\end{aligned} \tag{3.7}$$

Proposition 4. *In the symmetric contest where the winning prize $T_1 = T_2$, there exists a unique symmetric mixed strategies equilibrium in the rent seeking game, where two identical firms choose the low investment in CSR projects with probability λ that is given by*

$$\lambda = (\delta_2 - \delta_1)/\delta_2$$

and apply the mixed strategies in effort that are given by

$$F_H(e) = \frac{e\delta_2}{\delta_1 T(1-\delta_2)}$$

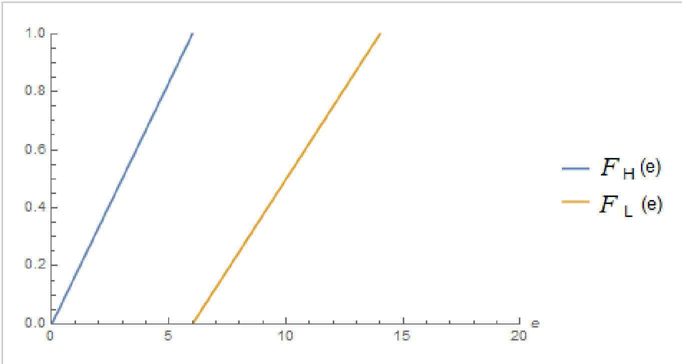
and

$$F_L(e) = \frac{\delta_1 T - \delta_2(e + \delta_1 T)}{(\delta_1 - \delta_2)T}.$$

As a numerical example, we set $T = 20, \delta_1 = 0.3, \delta_2 = 0.5$ and report the effort distributions conditional on the investments in Figure 3.1. It shows that high investments

Figure 3.1: Effort distributions associated with High and Low CSR investment level and homogeneous winning prizes

$F_L(e), F_H(e)$



(in blue) are associated with low efforts and low investments (in orange) are associated with high efforts. Firms uses mixed strategies in effort and the upper bound is far below the value of the winning prize 20. It suggests that with the introduction of the investment in CRS related matters, the lobbying costs are lower than that in a standard lobbying contest.

3.5 Asymmetric contests

In this section, we study asymmetric contests where two firms with different valuations for the winning prize compete for a contract. We consider a competition between a large (efficient) firm and a small (inefficient) firm. As in the symmetric case, both firms make a positive profit only when the authority has difficulty in choosing a more suitable winner, which is the case if at least one firm invests in CSR projects.

In a lobbying contest where $T_1 > T_2$, denote the effort strategies of firm 1 and firm 2 by F_1 and F_2, respectively. These cumulative distribution functions $F_i(e_i)$ are increasing in $e_i > 0$ and continuous in the support $[\underline{e_i}, \overline{e_i}]$, for firm $i \in \{1, 2\}$. If there exists any equilibrium, as proved in Lemma 8, the equilibrium must have the following properties.

Lemma 9 (Asymmetric Contest)**.**

(1) there exists $e_i^\star \in [\underline{e_i}, \overline{e_i}]$ *s.t.* $c_i = L$ *for* $e_i \geq e_i^\star$ *and* $c_i = H$ *for* $e_i < e_i^\star, i \in \{1, 2\}$

(2) $\underline{e_1} = \underline{e_2} = 0$

(3) $\overline{e_1} = \overline{e_2} > 0$

(4) $F_1(0)F_2(0) = 0$

The first item in the lemma tells that the firm with high effort would choose low investment and vice versa. For firm i that chooses low investment, the effort is in the range of $[\underline{e_i}, e_i^\star)$ and for high investment, is in the range of $[e_i^\star, \overline{e_i}]$. When two firm chooses the investment levels $(c_1, c_2) = (L, L)$, both obtain the expected profits

$$T_i F_j(e_i) - e_i, i \neq j$$

where the lower bound $\underline{e_i}$ for both firms must be zero as in standard all pay auctions. Otherwise we can easily find profitable deviation. When two firms choose the investment levels $(c_1, c_2) = (H, H)$, both obtain the expected profit

$$T_i \delta_2/2 + T_i(1 - \delta_2)F_j(e_i) - e_i, i \neq j$$

The upper bound $\overline{e_i}$ for both firms must be the same. Otherwise one firm can deviate to a slight higher effort level e' that ensures a higher expected profit by $F(e') = 1$. The last lemma tells that one and only one firm would choose zero effort with zero possibility. We will soon show that it is the firm with high valuation. And for its competitor, the lower the valuation is, the more unlikely it is to choose zero effort.

We construct an equilibrium that satisfies the above lemmas. We then show the equilibrium uniqueness by ruling out other candidate equilibria. Start from the case where $e_1^\star \leq e_2^\star$. This entails that the profits of firm 2, evaluated at different points, are given by

$$\Pi_2 = T_2[F_1(e_2)\left(\delta_2/2 + (1 - \delta_2)\right) + \left(F_1(e_1^\star) - F_1(e_2)\right)\left(\delta_2/2\right) + \left(1 - F_1(e_1^\star)\right)\left(\delta_1/2\right)] - e_2, \quad \text{for } e_2 < e_1^\star \tag{3.8}$$

$$\begin{aligned}\Pi_2 = T_2[&F_1(e_1^\star)\left(\delta_2/2 + (1 - \delta_2)\right) + \left(F_1(e_2) - F_1(e_1^\star)\right)\left(\delta_1/2 + (1 - \delta_1)\right) \\ &+ \left(1 - F_1(e_2)\right)\left(\delta_1/2\right)] - e_2, \\ &\text{for } e_2 \in [e_1^\star, e_2^\star)\end{aligned} \tag{3.9}$$

$$\Pi_2 = T_2[F_1(e_1^\star)\,(\delta_1/2 + (1-\delta_1)) + (F_1(e_2) - F_1(e_1^\star))\,(\delta_0/2 + (1-\delta_0))$$
$$+ (1 - F_1(e_2))\,(\delta_0/2)] - e_2,$$
$$\text{for } e_2 \geq e_2^\star \tag{3.10}$$

Likewise, the profit of firm 1 evaluated at different points of the support of the effort distribution is given by

$$\Pi_1 = T_1[F_2(e_1)\,(\delta_2/2 + (1-\delta_2)) + (F_2(e_2^\star) - F_2(e_1))\,(\delta_2/2) + (1 - F_2(e_2^\star))\,(\delta_1/2)] - e_1,$$
$$\text{for } e_1 < e_1^\star \tag{3.11}$$

$$\Pi_1 = T_1[F_2(e_1)\,(\delta_1/2 + (1-\delta_1)) + (F_2(e_2^\star) - F_2(e_1))\,(\delta_1/2) + (1 - F_2(e_2^\star))\,(\delta_0/2)] - e_1,$$
$$\text{for } e_1 \in [e_1^\star, e_2^\star) \tag{3.12}$$

$$\Pi_1 = T_1[F_2(e_2^\star)\,(\delta_1/2 + (1-\delta_1)) + (F_2(e_1) - F_2(e_2^\star))\,(\delta_0/2 + (1-\delta_0))$$
$$+ (1 - F_2(e_1))\,(\delta_0/2)] - e_1,$$
$$\text{for } e_1 \geq e_2^\star \tag{3.13}$$

To solve for $e_i^\star$, we note that the equilibrium profits should be the same at $e_i^\star - \epsilon$ and $e_i^\star + \epsilon$, where $\epsilon > 0$ and $\epsilon \to 0$. Equating the profits of firm 2 in Equations (3.9) and (3.10) results in

$$\begin{aligned}
&F_1(e_1^\star)\,(\delta_2/2 + (1-\delta_2) - \delta_1/2 - (1-\delta_1)) \\
&+ (F_1(e_2^\star) - F_1(e_1^\star))\,(\delta_1/2 + (1-\delta_1) - \delta_0/2 - (1-\delta_0)) \\
&+ (1 - F_1(e_2^\star))\,(\delta_1/2 - \delta_0/2)] = 0
\end{aligned} \tag{3.14}$$

where $F_1(e_1^\star)$ is given by Equation (3.9)

$$\Pi_2 = T_2[F_1(e_1^\star)\,(\delta_2/2 + (1-\delta_2)) + (1 - F_1(e_1^\star))\,(\delta_1/2)] - e_1^\star$$
$$\Rightarrow F_1(e_1^\star) = \frac{-2e_1^\star - 2\Pi_2 + \delta_1 T_2}{(-2+\delta_1+\delta_2)T_2},$$

and $F_1(e_2^\star)$ is given by Equation (3.10)

$$\begin{aligned}\Pi_2 &= T_2[F_1(e_1^\star)\,(\delta_1/2 + (1-\delta_1)) \\ &\quad + (F_1(e_2^\star) - F_1(e_1^\star))\,(\delta_0/2 + (1-\delta_0)) \\ &\quad + (1 - F_1(e_2^\star))\,(\delta_0/2)] - e_2^\star \\ \Rightarrow F_1(e_2^\star) &= \frac{2e_2^\star + 2\Pi_2 - \delta_1 T_2 - \delta_1 F_1(e_1^\star) T_2 + \delta_2 F_1(e_1^\star) T_2}{2T_2 - 2\delta_1 T_2}.\end{aligned}$$

Equating the profits of firm 1 in Equations (3.11) and (3.12) gives

$$\begin{aligned}&F_2(e_1^\star)\,(\delta_2/2 + (1-\delta_2) - \delta_1/2 - (1-\delta_1)) \\ &+ (F_2(e_2^\star) - F_2(e_1^\star))\,(\delta_2/2 - \delta_1/2) \\ &+ (1 - F_2(e_2^\star))\,(\delta_1/2 - \delta_0/2) = 0 \end{aligned} \tag{3.15}$$

where $F_2(e_2^\star)$ is given by Equation (3.13)

$$\begin{aligned}\Pi_1 &= T_1[F_2(e_2^\star)\,(\delta_1/2 + (1-\delta_1)) + (1 - F_2(e_2^\star))\,(\delta_0/2)] - e_2^\star \\ \Rightarrow F_2(e_2^\star) &= -\frac{2(e_2^\star + \Pi_1)}{(-2+\delta_1)T_1},\end{aligned}$$

and $F_2(e_1^\star)$ is given by Equation (3.12)

$$\begin{aligned}\Pi_1 &= T_1[F_2(e_1^\star)\,(\delta_1/2 + (1-\delta_1)) + (F_2(e_2^\star) - F_2(e_1^\star))\,(\delta_1/2) + (1 - F_2(e_2^\star))\,(\delta_0/2)] - e_1^\star \\ \Rightarrow F_2(e_1^\star) &= \frac{2e_1^\star + 2\Pi_1 - \delta_1 T_1 + \delta_1 F_2(e_2^\star) T_1 - \delta_2 F_2(e_2^\star) T_1}{2T_1 - 2\delta_2 T_1}.\end{aligned}$$

By $\delta_0 = 0$, solving the system of linear equations (3.14) and (3.15), we get the solution for $e_1^\star$ and $e_2^\star$.

$$\begin{aligned}e_1^\star &= e_1^\star(T_1, T_2, \Pi_1, \Pi_2, \delta_1, \delta_2) \\ &= \{2\delta_2^2\Pi_2 - 2\delta_1\delta_2[(-2+\delta_2)\Pi_1 + 2\Pi_2 + T_2] + \delta_1^4(-T_1 + T_2) - \delta_1^3[(-4+\delta_2)T_1 + 4T_2] \\ &\quad + \delta_1^2(-4T_1 + 4T_2 + \delta_2(-2\Pi_1 + 2\Pi_2 + 2T_1 + T_2))\}/\{2(-1+\delta_1)\delta_2^2\}\end{aligned} \tag{3.16}$$

$$
\begin{aligned}
e_2^\star &= e_2^\star(T_1, T_2, \Pi_1, \Pi_2, \delta_1, \delta_2) \\
&= \{-2\delta_2^2(\Pi_1 - 2\Pi_2) + \delta_1^4(-T_1 + T_2) - 2\delta_1\delta_2(-2\Pi_1 + (2+\delta_2)\Pi_2 - T_1 + 2T_2) \\
&+ \delta_1^2(-4T_1 - \delta_2(2\Pi_1 - 2\Pi_2 + T_1 - 4T_2) + 4T_2) \\
&+ \delta_1^3(4T_1 - (4+\delta_2)T_2)\}/[2(-1+\delta_1)\delta_2^2] \qquad (3.17)
\end{aligned}
$$

We also know that the equilibrium profits must be the same at $e_i = 0$ and at $e_i = \overline{e}$. For firm 1, this yields

$$
\begin{aligned}
&\Pi_1(0) = \Pi_1(\overline{e}) \\
\Rightarrow\ & T_1(\frac{1}{2}\delta_1(1 - F_2(e_2^\star)) + \frac{1}{2}\delta_2(F_2(e_2^\star) - F_2(0)) + (1 - \delta_2/2)F_2(0)) \\
&= T_1((1 - \delta_0/2)(1 - F_2(e_2^\star)) + (1 - \delta_1/2)F_2(e_2^\star)) - \overline{e} \qquad (3.18)
\end{aligned}
$$

And for firm 2, we have

$$
\begin{aligned}
&\Pi_2(0) = \Pi_2(\overline{e}) \\
\Rightarrow\ & T_2[\frac{1}{2}\delta_1(1 - F_1(e_1^\star)) + \frac{1}{2}\delta_2(F_1(e_1^\star) - F_1(0)) + (1 - \delta_2/2)F_1(0)] \\
&= T_2[(1 - \delta_0/2)(1 - F_1(e_1^\star)) + (1 - \delta_1/2)F_1(e_1^\star)] - \overline{e} \qquad (3.19)
\end{aligned}
$$

By $\delta_0 = 0$, we simply the last equation.

$$
2\overline{e} + (-2 + \delta_1 + \delta_2 F_1(e_1^\star) + 2F_1(0) - 2\delta_2 F_1(0))T_2 = 0 \qquad (3.20)
$$

Knowing that Lemma 9 must hold in equilibrium, we show that there is an equilibrium under the conditions $F_1(0) = 0$ and $F_2(0) > 0$.[16] Equation (3.20) is further simplified as

$$
\overline{e} = -\frac{1}{2}(-2 + \delta_1 + \delta_2 F_1(e_1^\star))T_2. \qquad (3.21)
$$

[16]We check the existence of equilibrium under three other conditions: 1) $e_1^\star \leq e_2^\star$, $F_1(0) > 0$ and $F_2(0) = 0$; 2) $e_1^\star > e_2^\star$, $F_1(0) > 0$ and $F_2(0) = 0$; and 3) $e_1^\star > e_2^\star$, $F_1(0) = 0$ and $F_2(0) > 0$. We prove that there is no equilibrium in these cases. The proofs are available upon request.

Plugging this into Equation (3.18), we get

$$F_2(0) = \frac{-2T_1 + \delta_0 T_1 + \delta_1 T_1 - \delta_0 F_2(e_2^\star)T_1 + \delta_2 F_2(e_2^\star)T_1 + 2T_2 - \delta_1 T_2 - \delta_2 F_1(e_1^\star)T_2}{2(-1+\delta_2)T_1} \tag{3.22}$$

We plug Equations (3.16), (3.17), (3.21) and (3.22) into (3.14) and (3.15), and get the solution to the equilibrium profits Π_1 and Π_2

$$\Pi_1 = [1 - 2\delta_1 - \delta_1^3/\delta_2^2 + (5\delta_1^2)/(2\delta_2)]T_1 + [-1 + 3\delta_1 + \delta_1^3/\delta_2^2 - (3\delta_1^2)/\delta_2]T_2 \tag{3.23}$$

$$\Pi_2 = -\frac{\delta_1[2\delta_2^2(T_1 - 2T_2) + 2\delta_1^2(T_1 - T_2) + \delta_1\delta_2(-4T_1 + 5T_2)]}{2\delta_2^2} \tag{3.24}$$

and

$$e_1^\star = \frac{\delta_1(-1+\delta_2)(-2\delta_1 T_1 + 2\delta_2 T_1 + 2\delta_1 T_2 - 3\delta_2 T_2)}{\delta_2^2} \tag{3.25}$$

$$e_2^\star = T_1 - \frac{\delta_1^2(-2+\delta_2)(T_1 - T_2)}{\delta_2^2} - T_2 + \frac{\delta_1((-3+\delta_2)T_1 - 2(-2+\delta_2)T_2)}{\delta_2} \tag{3.26}$$

$$F_2(e_2^\star) = \frac{2\delta_1^2(T_1 - T_2) + 2\delta_2^2(T_1 - T_2) + \delta_1\delta_2(-3T_1 + 4T_2)}{\delta_2^2 T_1} \tag{3.27}$$

$$F_1(e_1^\star) = \frac{\delta_1(2\delta_1 T_1 - 2\delta_2 T_1 - 2\delta_1 T_2 + 3\delta_2 T_2)}{\delta_2^2 T_2} \tag{3.28}$$

$$F_2(e_1^\star) = \frac{2\delta_1^2(T_1 - T_2) + \delta_2^2(T_1 - T_2) + \delta_1\delta_2(-2T_1 + 3T_2)}{\delta_2^2 T_1} \tag{3.29}$$

$$F_1(e_1^\star) = \frac{\delta_1(2\delta_1 T_1 - 2\delta_2 T_1 - 2\delta_1 T_2 + 3\delta_2 T_2)}{\delta_2^2 T_2} \tag{3.30}$$

Therefore, plugging Equations (3.23)–(3.30) into (3.8) and (3.11), we get the effort distributions for the high valuation firm

$$F_1(e) = \begin{cases} \frac{e}{(1-\delta_2)T_2} & \text{for } e < e_1^\star \\ \frac{2\delta_1^3(T_1-T_2)+\delta_1^2\delta_2(-4T_1+5T_2)-\delta_2^2(e-2\delta_1T_1+3\delta_1T_2)}{(-1+\delta_1)\delta_2^2T_2} & \text{for } e_1^\star \le e < e_2^\star \\ \frac{\delta_1^2(T_1-T_2)+\delta_2(e-\delta_1T_1+2\delta_1T_2)}{\delta_2T_2} & \text{for } e \ge e_2^\star \end{cases} \tag{3.31}$$

and for the low valuation firm

$$F_2(e) = \begin{cases} 1 + \frac{e}{T_1-\delta_2 T_1} - \frac{T_2}{T_1} & \text{for } e < e_1^\star \\ \frac{2\delta_1^3(T_1-T_2)+\delta_1^2\delta_2(-4T_1+5T_2)-\delta_2^2(e+T_1-3\delta_1 T_1-T_2+4\delta_1 T_2)}{(-1+\delta_1)\delta_2^2 T_1} & \text{for } e_1^\star \le e < e_2^\star \\ \frac{\delta_1^2(T_1-T_2)+\delta_2(e+T_1-\delta_1 T_1-T_2+2\delta_1 T_2)}{\delta_2 T_1} & \text{for } e \ge e_2^\star \end{cases} \tag{3.32}$$

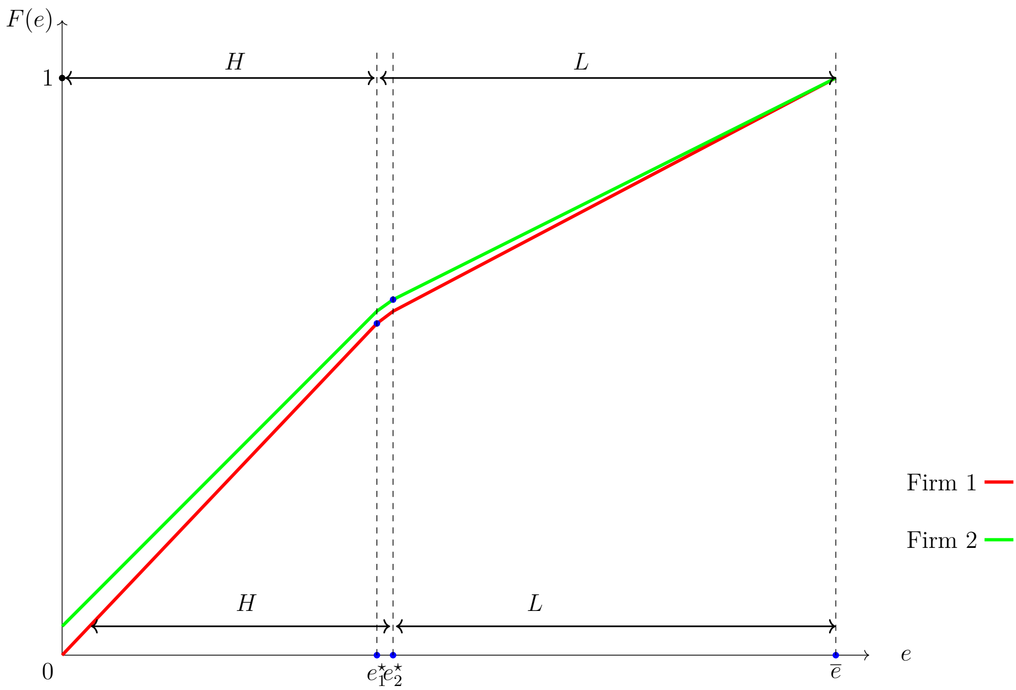
As a numerical example, we set $T_1 = 20, T_2 = 19, \delta_1 = 0.3, \delta_2 = 0.5$. We get the effort distributions for e_1 and e_2 as illustrated in figure 3.2. The effort distributions $F_1(\cdot)$ and $F_2(\cdot)$ monotonically increase from zero to the upper bound $\overline{e} = 13.42$ which is much lower than both firm's valuations. in the interval $[\underline{e_1}, e_1^\star) = [0, 5.46)$, firm 1 selects high investment; otherwise it selects low. In the interval $[\underline{e_2}, e_2^\star) = [0, 5.75)$, firm 2 selects high investment and low otherwise. Furthermore, firm 2, which profits less from winning, spends nothing with a strictly positive probability. An increase in T_2 would change the graphs in two ways: 1) Firm 2's probability of spending nothing $F_2(0)$ decreases. 2) The cutoff point $e_2^\star$ moves towards $e_1^\star$. In the case where two firms have the same valuations $T_1 = T_2$, the two graphs overlap, as they should.

We find that the more efficient firm lobbies more and invests less in CSR.This is intuitive once we observe that the more efficient firm gains more from winning. Higher payoff makes the firm compete more strongly, increasing its probability of winning the contract. Lobbying is a more competitive PR strategy than CSR investment in our game. In equilibrium, the firm faces a tradeoff between efficiency (winning more often) and rent extraction/dissipation (saving on costs) which it has to solve in choosing how much to invest in CSR.

We prove the equilibrium uniqueness in three steps. First, we find the support of candidate equilibria. Second, in the support, we construct one equilibrium. Third, holding the basic settings unchanged, we rule out other possibilities. Since we have already completed the first two steps and prove the existence of an mixed strategy equilibrium. Now we check other candidate equilibria. More precisely, We duplicate the above analysis and test the equilibrium existence under following conditions: 1) $e_1^\star \le e_2^\star$, $F_1(0) > 0$ and $F_2(0) = 0$; 2) $e_1^\star > e_2^\star$, $F_1(0) > 0$ and $F_2(0) = 0$; and 3) $e_1^\star > e_2^\star$, $F_1(0) = 0$ and $F_2(0) > 0$. In all these cases, there exists contradiction.[17] Thus we prove there is no other equilibrium in the support $[0, \overline{e}]$. The equilibrium we constructed

[17] The analysis is completed with the help of Mathematica and available upon request.

Figure 3.2: effort distribution associated with High and Low investment and asymmetric winning prize



above is the unique mixed strategy equilibrium in the support.

3.6 Discussion

Next, we discuss some immediate extensions of our analysis.

3.6.1 Procurement with Competition in Prices

So far we have assumed that firms compete by spending money in lobbying or CSR to make themselves more competitive in a market for government contracts. Yet, one of the main motivations for using a public tender to determine the recipient of a government contract is that of economizing on costs of provision. In those cases, a firm's proposal usually consists of a price offer p_i together with some background information on delivery conditions and investments in CSR, etc.

Then, we can still think that a firm chooses how much to invest in winning the contest, i.e, e, and then whether to invest this (i) into something that is easy for the authority to compare, i.e. by offering a low p_i, and (ii) into something that is hard for the authority to compare, i.e., by investing in CSR. By the same logic as before, investment in CSR relaxes competition and increases prices.

However, because a lower contracting price may allow the government to allocate the money saved to more productive uses, in terms of welfare the setting is very different from a lobbying contest where firms' investments are inefficient. Thus, the government faces a tradeoff between the price and the importance of the CSR investment. Depending on which is more efficient, CSR investment by firms or government use of money, socially responsive procurement can either increase or decrease welfare.

3.6.2 Contests as Markets for Influence

Fudenberg and Tirole (1987) exemplify how the hypothesis of full rent dissipation in a contest for a monopoly strongly depends on the specific extensive form of the game, e.g., contestable markets, war of attrition competition, capacity commitments, etc., which determines both rent dissipation and the wastefulness of expenditure. The importance of specifying the strategic variables available to firms is also stressed by Menezes and Quiggin (2010). They demonstrate that the standard Tullock solution corresponds to an oligopolistic market equilibrium where firms choose market shares. Previously, Baye et al. (1996) show that all-pay auctions are isomorphic to models like Varian (1980) for oligopolistic competition with imperfect information.

Thus, Menezes and Quiggin (2010) argue that being explicit about the strategic variables available to firms is essential to understand rent dissipation in a contest because the mode of competition determines what they call *the price of influence*, i.e., the expenditure required to receive the prize. Our paper is an important addition to this discussion as it describes a setup where firms themselves can determine the intensity of competition in the market for government contracts by choosing whether to lobby (the more competitive option) or invest in CSR (the less competitive option). We observe that firms have incentives to differentiate and obfuscate their lobbying strategies in order to relax competition and lower the price they need to pay to influence the contest.

3.6.3 Desirability of Transparency in Lobbying

Our analysis can also be used to shed more light on the desirability of transparency in lobbying. Many public acts seek to make lobbying spending more transparent. The EU transparency register for lobbyists is just one such example. Not without reason, lobbying is often viewed as a shady realm. The chains of influence in networks of political connections are typically complicated and the strategies with which interest groups try to influence decisions can be innovative.[18]

The shadiness could obviously serve both good and bad purposes. First, by hiding chains of influence from public eyes and allowing secret dealings, it might allow private interest groups to bribe politicians and regulators. That is obviously not desirable. Second, it could make lobbying expenditures/political contributions harder to compare, e.g., if they are very different in nature or if they come through different sources. The effects of this kind of shadiness may not be that worrisome.

As discussed in the earlier section, lobbying is just a form of selling: not to consumers but to politicians or regulators. As in consumer markets, lobbyists have incentives to economize on costs by obfuscating their lobbying strategies. We have shown that, in so far that lobbying is wasteful, this is actually welfare improving. In other words, as long as the shadiness reduces lobbying competition, it should not be a concern for a benevolent politician. Then, only the politician who is afraid of losing his/her bribes should be worried. Interestingly, statistics show that lobbying expenditure doubled in 1998–2008, from \$1.45 to \$3.3 Billion in the US, at the same time as technological progress increased transparency.

3.7 Conclusion

In this paper, we study how socially responsible procurement affects the money that firms allocate to influence the outcome of a public procurement contest, modeled as an all-pay auction. This is a topical question because different authorities all over the world have recently established guidelines for socially responsible procurement. In the model, firms can try to increase their competitiveness in the market for government contracts

[18] As described by Vining et al. (2005), a firm's PR or lobbying strategy is a multi-dimensional object comprising the choice of (1) the jurisdictional venue(s); (2) the organizational target(s), (3) cooperation between lobbies, (4) the argument(s) used, (5) the delivery mode(s), etc.

both directly by lobbying and by investing in corporate social responsibility (CSR). CSR investments could take many forms (e.g., corporate governance vs. environmental investments) making them harder to compare for the government authority than lobbying spending. As a result, we find that CSR can act as an effective differentiation strategy for firms competing for a contract.

In particular, the paper shows that socially responsible procurement (i) alleviates the competition among firms for government contracts, (ii) shifts firms' public relations spending from lobbying to CSR investment, and (iii) decreases the total amount of money that a firm spends to influence the authority. This is welfare improving in so far that lobbying spending is socially wasteful. The analysis also helps to weigh the pros and cons of socially responsible procurement when a public tender is made to reduce public spending, rationalizes firms' incentives to differentiate their PR and lobbying strategies, and sheds new light on the desirability and beneficiaries of transparency in lobbying. Socially responsible procurement may elevate public spending and transparency may increase rents to political office.

Appendix: Analysis of Winning Probabilities

We measure the winning probability in the asymmetric case and compare it with a standard asymmetric all-pay-auction. In a standard asymmetric all-pay-auction where firm 1 has a higher winning prize, the firms' equilibrium strategies are

$$F_1(e) = e/T_2$$
$$F_2(e) = 1 - T_2/T_1 + e/T_1$$

where the winning probabilities are

$$W_1 = \int_0^{T_2} (1 - T_2/T_1 + e/T_1) d(e/T_2) = 1 - \frac{T_2}{2T_1}$$
$$W_2 = \int_0^{T_2} (e/T_2) d(1 - T_2/T_1 + e/T_1) = \frac{T_2}{2T_1}.$$

Now we find the winning probabilities when firms can choose CSR investment and lobbying simultaneously.

Firm 1's winning probability is given by

$$
\begin{aligned}
WP_1 = & \int_0^{e_1^\star} \left\{ (1-\delta_2)F_2(e) + \delta_2 \frac{1}{2} F_2(e_2^*) + \delta_1 \frac{1}{2}[F_2(\overline{e}) - F_2(e_2^*)] \right\} dF_1(e) \\
& + \int_{e_1^\star}^{e_2^\star} \left\{ (1-\delta_1)F_2(e) + \delta_1 \frac{1}{2} F_2(e_2^*) + \delta_0 \frac{1}{2}[F_2(\overline{e}) - F_2(e_2^*)] \right\} dF_1(e) \\
& + \int_{e_2^\star}^{\overline{e}} \left\{ (1-\delta_0)[F_2(e) - F_2(e_2^*)] + (1-\delta_1)F_2(e_2^*) + \delta_0 \frac{1}{2}[F_2(\overline{e}) - F_2(e_2^*)] + \delta_1 \frac{1}{2} F_2(e_2^*) \right\} dF_1(e) \\
& = \int_0^{e_1^\star} \left\{ (1-\delta_2)(1 + \frac{e}{T_1 - \delta_2 T_1} - \frac{T_2}{T_1}) + \delta_2 \frac{1}{2} (\frac{2\delta_1^2(T_1-T_2) + 2\delta_2^2(T_1-T_2) + \delta_1\delta_2(-3T_1+4T_2)}{\delta_2^2 T_1}) \right. \\
& \quad \left. + \delta_1 \frac{1}{2}[1 - (\frac{2\delta_1^2(T_1-T_2) + 2\delta_2^2(T_1-T_2) + \delta_1\delta_2(-3T_1+4T_2)}{\delta_2^2 T_1})] \right\} d\frac{e}{(1-\delta_2)T_2} \\
& + \int_{e_1^\star}^{e_2^\star} \left\{ (1-\delta_1)(\frac{2\delta_1^3(T_1-T_2) + \delta_1^2\delta_2(-4T_1+5T_2) - \delta_2^2(e + T_1 - 3\delta_1 T_1 - T_2 + 4\delta_1 T_2)}{(-1+\delta_1)\delta_2^2 T_1}) \right. \\
& \quad \left. + \delta_1 \frac{1}{2} (\frac{2\delta_1^2(T_1-T_2) + 2\delta_2^2(T_1-T_2) + \delta_1\delta_2(-3T_1+4T_2)}{\delta_2^2 T_1}) \right\} \\
& \qquad d(\frac{2\delta_1^3(T_1-T_2) + \delta_1^2\delta_2(-4T_1+5T_2) - \delta_2^2(e - 2\delta_1 T_1 + 3\delta_1 T_2)}{(-1+\delta_1)\delta_2^2 T_2}) \\
& + \int_{e_2^\star}^{\overline{e}} \left\{ (1-\delta_0)[(\frac{\delta_1^2(T_1-T_2) + \delta_2(e + T_1 - \delta_1 T_1 - T_2 + 2\delta_1 T_2)}{\delta_2 T_1}) \right. \\
& \qquad - (\frac{2\delta_1^2(T_1-T_2) + 2\delta_2^2(T_1-T_2) + \delta_1\delta_2(-3T_1+4T_2)}{\delta_2^2 T_1})] \\
& \qquad + (1-\delta_1)(\frac{2\delta_1^2(T_1-T_2) + 2\delta_2^2(T_1-T_2) + \delta_1\delta_2(-3T_1+4T_2)}{\delta_2^2 T_1}) \\
& \qquad \left. + \delta_1 \frac{1}{2} (\frac{2\delta_1^2(T_1-T_2) + 2\delta_2^2(T_1-T_2) + \delta_1\delta_2(-3T_1+4T_2)}{\delta_2^2 T_1}) \right\} \\
& \qquad d(\frac{\delta_1^2(T_1-T_2) + \delta_2(e - \delta_1 T_1 + 2\delta_1 T_2)}{\delta_2 T_2}).
\end{aligned}
$$

Firm 2's winning probability is given by

$$
\begin{aligned}
WP_2 = & \int_0^{e_1^\star} \left\{(1-\delta_2)F_1(e) + \delta_2 \frac{1}{2} F_1(e_1^*) + \delta_1 \frac{1}{2}[F_1(\overline{e}) - F_1(e_1^*)]\right\} dF_2(e) \\
& + \int_{e_1^\star}^{e_2^\star} \left\{(1-\delta_2)F_1(e_1^*) + (1-\delta_1)[F_1(e) - F_1(e_1^*)] + \delta_1 \frac{1}{2}[F_1(\overline{e}) - F_1(e_1^*)] + \delta_2 \frac{1}{2} F_1(e_1^*)\right\} dF_2(e) \\
& + \int_{e_2^\star}^{\overline{e}} \left\{(1-\delta_0)[F_1(e) - F_1(e_1^*)] + (1-\delta_1)F_1(e_1^*) + \delta_0 \frac{1}{2}[F_1(\overline{e}) - F_1(e_1^*)] + \delta_1 \frac{1}{2} F_1(e_1^*)\right\} dF_2(e) \\
= & \int_0^{e_1^\star} \left\{(1-\delta_2)(\frac{e}{(1-\delta_2)T_2}) + \delta_2 \frac{1}{2}(\frac{\delta_1(2\delta_1 T_1 - 2\delta_2 T_1 - 2\delta_1 T_2 + 3\delta_2 T_2)}{\delta_2^2 T_2})\right. \\
& \left. + \delta_1 \frac{1}{2}[1 - (\frac{\delta_1(2\delta_1 T_1 - 2\delta_2 T_1 - 2\delta_1 T_2 + 3\delta_2 T_2)}{\delta_2^2 T_2})]\right\} d(1 + \frac{e}{T_1 - \delta_2 T_1} - \frac{T_2}{T_1}) \\
& + \int_{e_1^\star}^{e_2^\star} \left\{(1-\delta_2)(\frac{\delta_1(2\delta_1 T_1 - 2\delta_2 T_1 - 2\delta_1 T_2 + 3\delta_2 T_2)}{\delta_2^2 T_2})\right. \\
& + (1-\delta_1)[(\frac{2\delta_1^3(T_1 - T_2) + \delta_1^2\delta_2(-4T_1 + 5T_2) - \delta_2^2(e - 2\delta_1 T_1 + 3\delta_1 T_2)}{(-1+\delta_1)\delta_2^2 T_2}) \\
& - (\frac{\delta_1(2\delta_1 T_1 - 2\delta_2 T_1 - 2\delta_1 T_2 + 3\delta_2 T_2)}{\delta_2^2 T_2})] + \delta_1 \frac{1}{2}[1 - (\frac{\delta_1(2\delta_1 T_1 - 2\delta_2 T_1 - 2\delta_1 T_2 + 3\delta_2 T_2)}{\delta_2^2 T_2})] \\
& \left. + \delta_2 \frac{1}{2}(\frac{\delta_1(2\delta_1 T_1 - 2\delta_2 T_1 - 2\delta_1 T_2 + 3\delta_2 T_2)}{\delta_2^2 T_2})\right\} \\
& d(\frac{2\delta_1^3(T_1 - T_2) + \delta_1^2\delta_2(-4T_1 + 5T_2) - \delta_2^2(e + T_1 - 3\delta_1 T_1 - T_2 + 4\delta_1 T_2)}{(-1+\delta_1)\delta_2^2 T_1}) \\
& + \int_{e_2^\star}^{\overline{e}} \left\{(1-\delta_0)[\frac{\delta_1^2(T_1 - T_2) + \delta_2(e - \delta_1 T_1 + 2\delta_1 T_2)}{\delta_2 T_2} - (\frac{\delta_1(2\delta_1 T_1 - 2\delta_2 T_1 - 2\delta_1 T_2 + 3\delta_2 T_2)}{\delta_2^2 T_2})]\right. \\
& + (1-\delta_1)(\frac{\delta_1(2\delta_1 T_1 - 2\delta_2 T_1 - 2\delta_1 T_2 + 3\delta_2 T_2)}{\delta_2^2 T_2}) \\
& \left. + \delta_1 \frac{1}{2}(\frac{\delta_1(2\delta_1 T_1 - 2\delta_2 T_1 - 2\delta_1 T_2 + 3\delta_2 T_2)}{\delta_2^2 T_2})\right\} \\
& d(\frac{\delta_1^2(T_1 - T_2) + \delta_2(e + T_1 - \delta_1 T_1 - T_2 + 2\delta_1 T_2)}{\delta_2 T_1}).
\end{aligned}
$$

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Chapter 4

Non-organized Boycott: Alliance Advantage and Free Riding Incentives in Uneven Wars of Attrition [1]

4.1 Introduction

A firm lacking awareness of corporate social responsibility and deviating from generally accepted social norms may be boycotted by consumers (Fehr and Fischbacher (2004)). Boycotts can be launched and influenced by non-governmental organizations (NGOs) or, more commonly, non-organized voluntary consumer activism. [2]

Sometimes the boycotting group wins: the targeted firms are 'forced' to behave in a way that benefits the interests of the boycotters. However, surprisingly, most of

[1]This chapter is based on an article publish on the *Eurasian Economic Review* Zheng (2019).The author would like to thank the Editor and two anonymous reviewers.

[2]There are many underlying causes of boycott activity, for example, inappropriate promotion of infant formula in developing countries (Nestlé), war (limiting trade with Russia due to Ukraine), political reasons (US boycott of French wine 2003), protecting a river delta (Shell Oil), Soviet violations of human rights and invasion of Afghanistan (1980 Summer Olympics), abuse of monopoly power (Microsoft), unfair wages (Delano grape), production of genetically modified organisms (Monsanto), child labour (Nike and Lush's supply chain in Jharkand, India), animal rights (KFC), products with unacceptable slogans (Abercrombie & Fitch in USA), destroying agricultural farms, greenhouses, ancient olive groves (Caterpillar's D-9 bulldozers in Palestine), conserving energy and reducing carbon emissions (the Close the Door campaign in UK), continuing rise of food prices (Cottage cheese in Israel), climate change denial and lack of investment in renewable energy (Esso/ExxonMobil), Deepwater Horizon oil spill (BP) and more recently, location of factory (Oreo) and independence (SodaStream), just to list a few.

the time boycotts fail. This is so even when the number of participating boycotters is initially large. There are some clear difficulties for boycotts. First, a pure profit-driven firm may act without much concern for social norms. By being ‘patient’ enough, a firm can simply ignore boycotters’ behaviour and the potential damage to its corporate image and reputation as long as its financial loss remains at an acceptable level. Second, the free riding problem exists. Since the success of a boycott generally does not require the contribution of all consumers in the market, each self-interested individual thus, more or less, has incentives to free ride on the collective actions of other boycotters. Third, the non-consistency of boycotters’ behaviour limits the financial and reputational damage to the misbehaving firm. In most of the cases, only very few consumers stop purchasing from the non-ethical firm ‘permanently’ (before boycott success is achieved). Many boycotters may not continuously fight against the firm for a long time, which limits the real boycotting power. Consumers may act as if they are ‘forgiving’ the unethical firm, which is in fact driven by selfishness and free ride incentives. Fourth, there are the non-commitment and non-binding features. Boycotting as a consumer’s independent choice is not under any binding obligations. It implies the great difficulty, in fact almost impossibility, of coordinating boycotts. Fifth, threatening to boycott has limited boycott power. Even if the boycotting is organized by some NGOs, it is more of a ‘threat to boycott’ rather than ‘physical boycotting activity’. The firm may view consumers’ threats as cheap talk. Last but not least, the ongoing boycott may turn out to be harmless. The consistency of a few consumers’ boycotting behavior does not necessarily achieve any good outcomes. An unethical firm can ignore this group — if there was indeed sufficient boycotting power that leads to a huge profit loss, the firm would have already changed its behavior in the first place. The firm only becomes more ‘patient’ and ‘persistent’ to fight against the ongoing boycotters over time with the hope of obtaining their potential ‘forgiveness’.

In this paper, we develop a non-cooperative game between a profit driven firm that lacks social responsibility and a number of potential boycotters who have environmental concerns. We try to address the following questions: on the demand side, which incentive dominates the consumer’s behaviour, moral concern (to boycott and do the right thing) or selfishness (not to boycott and focus on the subjective utility maximising)? Why do some consumers act like strict environmentalists (are willing to boycott with fighting costs for a long time) while others act like loyal supporters of the product (never participate in boycotts or boycott for a short period)? And on the supply side, does the size of firm, i.e., the number of potential boycotters that the firm serves, matter, and if so, to what extent?

Our model extends Maynard Smith (1974)'s classic two-contestant game of war of attrition. We introduce free riding incentives and alliance advantages to players on the demand side. Consumers form a weak alliance to fight against a misbehaving monopolist. Siding with the boycotters/alliance is cost-free but potentially beneficial. No commitment to fully cooperate is required. Meanwhile, a prize may be awarded to a free riding consumer even without providing a personal contribution. We allow a prize to a successful free rider if the game eventually ends with his ally's success. Each consumer wishes to obtain a prize with less effort.

We start from a two-against-one game. By comparing the mixed strategy equilibrium in our model to that of a classic two-contestant game, we examine the effect of the third player's intervention. We discuss how consumers behave in the presence of potential support from the alliance and how the firm deals with it. We then let the number of players on the demand side increase to infinitely large. We find a unique mixed strategy equilibrium in the n-against-one game. We derive optimal strategies for both sides at the limit.

Our results explain the difficulty of winning a non-organized boycott in reality. On the demand side, we find that the consumers' free riding incentives dominate their behaviours and therefore limit the real boycott power. The larger the market the firm serves, the more likely an individual consumer would surrender, which leaves fewer boycotters remaining in the costly conflict. We therefore explain the reason why some consumers act like loyal supporters of the product while others act like strict environmentalists. On the supply side, we show that the market size does not significantly affect the firm's strategies. The polluting firm would change its misbehaviour with slightly higher probability in a large economy. For a large monopolist that serves infinitely many consumers, boycott will surely be effective, that is, non-zero participation, but hardly successful, that is, lead to the firm's cessation of misbehaviour.[3]

Wars of attrition have been studied both theoretically and empirically by many

[3] In Craig Smith (1990), a boycott is 'effective' if there is non-zero participation and 'successful' if it leads to the cessation of the egregious act.

economists and social scientists after Maynard Smith (1974). However, only limited attention has been paid to the influence of a third contestant's contribution in a war of attrition framework taking account of alliance advantage and incentive to free ride. The closest studies to ours are Haigh and Cannings (1989); Bulow and Klemperer (1999) and Helgesson and Wennberg (2015), which discuss n players competing for one or several prizes. Powell (2017) discusses the third party effects on a two-player game by allowing a third party to take sides and provide endogenous intervention to one of the two actors. He studies the case where a third party chooses a time to support one side after a conflict starts. Therefore at the time of joining the game, the third party already forms a belief on the existing players' ability/strength ordering, the potential winning party and the expected length of the conflict. In this paper, we analyze a game within a game where the boycotters play a prisoners' dilemma against each other and meanwhile they form a weak alliance and play a war of attrition against the polluting firm.

Our research may also be linked with the literature on evolutionary games (see, e.g. Maynard Smith and Price (1973) and Laruelle et al. (2018)). Several papers studied boycotts, cooperation, punishment and environmental compliance (da Silva Rocha and Salomão (2019)) or more general punishment and cooperation games (da Silva Rocha et al. (2015) and da Silva Rocha (2017)) using an evolutionary game setting. In these papers rational behaviour is partially relaxed and the dynamic nature of the evolutionary game is analysed. In our paper, we employ a repeat game and assume players to be fully rational. Our model can be potentially extended to a two-population game where consumers' types and preferences change based on the interaction with other boycotters.

Several articles study consumer boycotts in different settings. Friedman (1991); Delacote (2008) provide conceptual discussion on boycott actions. Tyran and Engelmann (2005) provide an experiment on boycotts in reaction to a sudden cost increase in retail markets. They find that the cost increases the incidence of boycotts. Boycotts reduce market efficiency. Innes (2006) develops a model where two non-identical duopolists face a threat to boycott from an environmental organization. He finds that at equilibrium a small persistent boycott will work against the small firm or a large transitory boycott will work against the large firm. This implies larger firms are easier to defeat. Baron (2001) employs a game between an influential activist and a monopolist that is concerned about profit maximisation, altruism and activist's powerful threats. From a psychological perspective, John and Klein (2003) explain consumers' boycotting incentives and willingness to sacrifice. Heijnen and van der Made (2012) find that in a market under asymmetric information where consumers can signal high moral values, consumers always boycott with positive probability despite free-riding incentives and eventually result in a firm altering its behaviour. In a war of attrition framework, Peck (2017) analyzes a game between a monopolist that produces two-period durable goods and consumers who demand a lower price. He derives both non-boycott equilibrium and boycott equilibria where a boycott occurs with positive probability. Egorov and Harstad (2017) develop a boycott game between a public regulator, a misbehaving firm and activists. They find that in a two-player game without the regulator, 'private politics' is beneficial for activists but harmful for firms. Meanwhile, in a three-player game, 'private politics' is harmful for activists but beneficial for firms. Our paper contributes to this literature by demonstrating that even if the benefit of free riding is very small, it is sufficient enough to undermine the probability of boycott success. We also discuss how market size affects a firm's decision making.

The rest of the article is organized as follows. In Section 4.2 we discuss the basic settings and underlying assumptions in a two-against-one game. The equilibrium results are in Section 4.3. In Section 4.4 we consider a general case where one firm fights against a side of many competitors. When the size increases to infinitely large, the equilibrium is derived at the limit. Section 4.5 concludes the paper.

4.2 Model

Consider three players in a two-sided game of war of attrition competing for the winning prize under complete information: two consumers P1 (he) and P2 (she) on the demand side and one monopolist P3 (it) on the supply side. Consumers form a weak alliance where individual rewards and losses are determined by their joint actions. Consumers are identical. They are assumed, unless stated otherwise, to have the same incentives and play symmetric strategies. Time is discrete. In each period, players move simultaneously. The action set is binary: $a_i \in \{F, S\}, i = 1, 2, 3$ where F and S represent fight and surrender, respectively. Players have the same prior. P3's action F is driven by lack of corporate social responsibility and therefore is viewed as amoral. The complementary action S matches the generally accepted social norm and therefore is viewed as moral.

On the demand side, on one hand, driven by moral concern, consumers attempt to defeat the firm and force it to change its behaviour. On the other hand, selfish motivation gives rise to a prisoners' dilemma. Each consumer wishes the other to contribute more, that is, to allow him/herself to free ride on the alliance benefit. On the supply side, the firm chooses the best response to the joint action of the consumers. It wins the conflict only if every consumer is defeated.[4]

The flow payoffs to the players are summarized in Table 4.1. In each period of a

Table 4.1: A summary of flow payoffs

State-2: one targeted firm VS a weak alliance of two consumers

	Active fighters	
	firm (*it*)	Each consumer (*she* or *he*)
Cost of fighting	$-m$ per active consumer	$-c$
Winning prize	v	w
Probability to quit	$q_{(2)}$	$p_{(2)}$

State-1: one targeted firm VS one remaining consumer

	Active fighters		
	firm (*it*)	Remainer (i)	Early quitter / free rider ($-i$)
Cost of fighting	$-m$	$-c$	0
Winning prize	v	w	ϵ
Probability to quit	$q_{(1)}$	$p_{(1)}$	1

[a] $v > 2m > 0$, $w > c > 0$, $w > \epsilon > 0$ eliminate 'instant surrender' equilibrium where either side or both sides do not bother to fight in the first place.
[b] State-n represents the state of the game where n demanders remain in the game actively.

two-against-one game, active fighters pay a cost: $-c$ to each consumer and $-m$ per consumer to the firm. Only one side can win the game and no tie is allowed. The winning prize is w to each consumer and v to the firm. In the case when all three players surrender at the same time, each consumer benefits from the firm's concession and gets a second winning prize ϵ.[5] Once one and only one consumer surrenders, suppose it is P2, she becomes an early quitter and is not allowed to return to the conflict. The game becomes a standard one-against-one war of attrition where the remainer P1 has

[4] One may argue that in reality the monopoly supplier's decision may depend on the preferences and actions of the majority consumers. Thus it would win (or lose) the game if, for example, half of the consumers surrender (or fight persistently). However, the selection of the winning cutoff point would not change the main results in this paper.

[5] The logic of the second winning prize comes from the unevenness of the game. Only consumers benefit from the supplier's concession in this case but not vice versa. Consider consumers stop boycotting and a polluting firm switches to a clean technology at the same time. Consumers receive the utility from public good (the clean environment) but not from their moral values (concern for social responsibility and joy from defeating the firm).

to fight alone against the firm. There is no more fighting cost to P2 after her surrender. Moreover, we allow her to receive a prize as alliance benefit. If the game eventually ends with her ally's success, she gets the second prize ϵ. If instead the firm is the final winner, the early quitter, along with the remaining consumer, gets zero. The common discount factor is $\delta \in (0, 1)$.

The game of war of attrition is as illustrated in Table 4.2. The game is played only

Table 4.2: A game of war of attrition among three players

(a) State-2: a 'two-against-one' game

P1(P2)		P3: F	P3: S	
	$F(F)$	$-c, (-c), -2m$	$w, w, 0$	$[1 - p_{(2)}][1 - p_{(2)}]$
	$F(S)$	$-c, (0), -m$	$w, (\epsilon), 0$	$[1 - p_{(2)}]p_{(2)}$
	$S(F)$	$0, (-c), -m$	$\epsilon, (w), 0$	$p_{(2)}[1 - p_{(2)}]$
	$S(S)$	$0, (0), v$	$\epsilon, (\epsilon), 0$	$p_{(2)}p_{(2)}$
		$1 - q_{(2)}$	$q_{(2)}$	

(b) State-1: a 'one-against-one' game

Remainer P1		Opponent P3: F	Opponent P3: S	
	F	$-c, (0), -m$	$w, (\epsilon), 0$	$1 - p_{(1)}$
	S	$0, (0), v$	$\epsilon, (\epsilon), 0$	$p_{(1)}$
		$1 - q_{(1)}$	$q_{(1)}$	

[a] The actions and payoffs of the other player (P2) in parentheses in Table (a).
[b] The payoffs to the early quitter in parentheses in Table (b).
[c] The stopping probabilities to P1, P2 and P3 are $p(n)$, $h(n)$ and $q(n)$, respectively, where $n = 1, 2$ represents the number of the remaining players on the demand side. P1 and P2 are assumed to play the same strategy unless stated otherwise.
[d] The mixed strategy equilibrium is $(p^*_{(2)}, p^*_{(2)}, q^*_{(2)})$ in State-2 if P1 and P2 play symmetric strategies. The mixed strategy equilibrium is $(p^*_{(1)}, 1, q^*_{(1)})$ in State-1 if P2 surrenders and the game continues as a 'one-against-one' war of attrition.
[e] At equilibrium, the continuation values to each active fighting consumer are $\alpha_1, \alpha_2, \alpha_3$. In State-1, the continuation values are α_1 to the remaining consumer and α_2 to the early quitter. At equilibrium, the continuation values to P3 are always zero and therefore not highlighted separately.

once. It stops immediately if either side surrenders or both sides surrender at the same time. Otherwise the game continues in State-2 if everyone remains, as Table 4.2a shows, or in State-1 if one of the consumers surrenders, as Table 4.2b shows.

4.3 Symmetric Equilibrium

We solve the game backwards. We start from State-1 where only one consumer remains in the game and fights against the firm alone. For a mixed strategy equilibrium to be possible, the indifference conditions for both active players, the remainer and the firm, should be satisfied. For the firm P3, in any time t, the utility of fighting for one more period $U_3(F)$ equals to that of surrendering immediately $U_3(S)$.

$$\begin{aligned} U_3(F) &= U_3(S) \\ \Rightarrow [-m + \beta_1\delta][1 - p_{(1)}] + vp_{(1)} &= 0 \end{aligned}$$

where the continuation value $\beta_1 = 0$ at equilibrium. Solving the equation we get

$$p^*_{(1)} = \frac{m}{m+v} \tag{4.1}$$

Suppose P2 is the early quitter. The remainer P1 is indifferent between staying in the conflict for one more period or quitting now.

$$\begin{aligned} U_1(F) &= U_1(S) \\ \Rightarrow [-c + \alpha_1\delta][1 - q_{(1)}] + wq_{(1)} &= \epsilon q_{(1)} \end{aligned}$$

where the continuation value

$$\alpha_1 = [1 - p_{(1)}]U_1(F) + p_{(1)}U_1(S) = \epsilon q_{(1)} > 0 \tag{4.2}$$

at equilibrium. Since the quitting probability $q_{(1)}$ is in the range of $(0, 1)$, we get the unique solution

$$\begin{aligned} q^*_{(1)} &= \frac{c - \alpha_1\delta}{c - \alpha_1\delta + w - \epsilon} = \frac{B - \sqrt{B^2 - 4c\delta\epsilon}}{2\delta\epsilon} \\ \Rightarrow U_1 = \alpha_1 &= \frac{B - \sqrt{B^2 - 4c\delta\epsilon}}{2\delta} \end{aligned} \tag{4.3}$$

where $B = c - \epsilon + \delta\epsilon + w > 0$. The continuation value α_1 measures the remainer's *desire to stay in the costly conflict*. A positive α_1 shows that the player is willing to fight continuously even without support from the alliance.

The early quitter P2 stops paying the fighting cost right after her concession. Meanwhile she may get the second winning prize ϵ in each period with probability $q_{(1)}$ which is the alliance's winning probability. The timing of receiving the prize can be any t from the period of her surrender to infinity. Therefore her expected utility is

$$\begin{aligned} U_2 &= (0+\epsilon)q_{(1)} + [1-q_{(1)}][1-p_{(1)}]\delta(0+\epsilon)q_{(1)} + ... \\ &= \frac{\epsilon q_{(1)}}{1-[1-q_{(1)}][1-p_{(1)}]\delta} \\ &=: \alpha_2 > 0 \end{aligned} \tag{4.4}$$

The expected utility to surrender measures *a consumer's incentive to free ride.* A positive α_2 shows that each consumer on the demand side wishes to free ride on the benefit of her/his ally's contribution. Moreover, we find that the following inequality always holds given that the firm plays a mixed strategy.

$$\alpha_2 > \alpha_1 > 0, \quad \forall q_{(1)} \in (0,1)$$

$$\Rightarrow \quad \overbrace{U_i(S)|_{a_{-i}=F} > U_i(F)|_{a_{-i}=S}}^{\text{incentive to free ride (selfishness)}} \hspace{-9em} \underbrace{\hspace{9em} > U_i(S)|_{a_{-i}=S} = 0}_{\text{incentive to fight (morality)}}, \quad i=1,2$$

The intuition of the last inequality is as follows. In the case where a win for the demand side does not require every one's contribution, each consumer hopes to be the only early quitter and obtains the highest utility $U_i(S)|_{a_{-i}=F}$, that is, when consumer i surrenders and the remaining ally fights alone. If a consumer failed to be the first quitter, she/he would stay in the conflict rather than follow the quitter and obtain the lower utility $U_i(F)|_{a_{-i}=S}$. However, the incentive to free ride may lead to an unpleasant result. If both players quit at the same time, they obtain zero utility. There is de facto 'no ride'. Both players may be better off by staying in the conflict longer.

The expected time of obtaining a winning prize $ED_{(1)}$ is the expected length of the

State-1 game.

$$
\begin{aligned}
ED_{(1)} &= 1\Big\{1-[1-q_{(1)}][1-p_{(1)}]\Big\} + 2[1-q_{(1)}][1-p_{(1)}]\Big\{1-[1-q_{(1)}][1-p_{(1)}]\Big\} + ... \\
&= \sum_{t=1}^{+\infty} t\Big\{[1-q_{(1)}][1-p_{(1)}]\Big\}^{t-1}\Big\{1-[1-q_{(1)}][1-p_{(1)}]\Big\} \\
&= \frac{1}{p_{(1)}[1-q_{(1)}]+q_{(1)}}
\end{aligned}
$$

Now we move to State-2 in which two consumers actively fight against the firm. Consumers' strategies are assumed to be stationary as long as the state of the game does not change, i.e., before any player concedes. We assume that identical consumers play symmetric strategies. Analogous to the above, we solve the indifference conditions. For the firm P3,

$$
\begin{gathered}
U_3(F) = U_3(S) \\
\Rightarrow [-2m+\beta_3\delta][1-p_{(2)}]^2 + [-m+\beta_1\delta][1-p_{(2)}]p_{(2)} + [-m+\beta_2\delta]p_{(2)}[1-p_{(2)}] \\
+v[p_{(2)}]^2 = 0
\end{gathered}
$$

where the continuation values βs $= 0$ at equilibrium. Since the quitting probability $p_{(2)}$ is in the range of $(0,1)$, we get a unique solution to the individual consumer's quitting probability

$$
p^*_{(2)} = \frac{-m+\sqrt{m^2+2mv}}{v} \tag{4.5}
$$

For active consumers Pi where $i = 1, 2$,

$$
\begin{gathered}
U_i(F) = U_i(S) \\
\Rightarrow [-c+\alpha_3\delta][1-p_{(2)}][1-q_{(2)}] + [-c+\alpha_1\delta]p_{(2)}[1-q_{(2)}] + wq_{(2)} \\
= [0+\alpha_2\delta][1-p_{(2)}][1-q_{(2)}] + \epsilon q_{(2)}
\end{gathered}
$$

where at equilibrium, the continuation value is

$$
\begin{aligned}
\alpha_3 &= [1-p_{(2)}]U_i(F) + p_{(2)}U_i(S) \\
&= \alpha_2\delta[1-p_{(2)}][1-q_{(2)}] + \epsilon q_{(2)}.
\end{aligned} \tag{4.6}
$$

We get

$$q^*_{(2)} = \frac{c - \delta[-\alpha_2 + \alpha_3 + (\alpha_1 + \alpha_2 - \alpha_3)p^*_{(2)}]}{c - \delta[-\alpha_2 + \alpha_3 + (\alpha_1 + \alpha_2 - \alpha_3)p^*_{(2)}] + w - \epsilon}. \tag{4.7}$$

Plugging in αs and $p^*_{(2)}$, we find a unique solution to $q^*_{(2)}$. The solution is reported in the appendix. Therefore in State-2, where two active boycotters remain in the boycott, there is a unique mixed strategy Nash equilibrium $(p^*_{(2)}, p^*_{(2)}, q^*_{(2)})$ where consumers are assumed to play symmetric stationary strategies.

Proposition 5. *There is a unique subgame perfect equilibrium such that each active boycotter surrenders with probability $p^*_{(n)}$ while the polluting firm surrenders with probability $q^*_{(n)}$ in State-n where $n \in \{1, 2\}$. In State-2 where two identical consumers actively fight against the polluting firm, consumers surrender with probability $p^*_{(2)}$, which is given by Equation (4.5), and the firm surrenders with probability $q^*_{(2)}$, which is given by Equation (4.7).*

The following results are derived immediately when we compare the equilibrium strategies in two states.

Lemma 10.

$$\begin{aligned}
&(1) \quad \frac{\partial p_{(2)}}{\partial \epsilon} = \frac{\partial p_{(1)}}{\partial \epsilon} = 0 \quad \textit{and} \\
& \quad \frac{\partial p_{(2)}}{\partial w} = \frac{\partial p_{(1)}}{\partial w} = 0 \\
&(2) \quad \frac{\partial p_{(2)}}{\partial \delta} = \frac{\partial p_{(1)}}{\partial \delta} = 0 \\
&(3) \quad p^*_{(2)} > p^*_{(1)} \quad \textit{and} \\
& \quad q^*_{(2)} < q^*_{(1)} \quad \textit{if} \quad \alpha_3 > \alpha_1 + \alpha_2
\end{aligned}$$

The first Lemma shows that, for the remaining consumer, the free rider's winning prize would not affect his own strategy at equilibrium. It is in line with our setting: players are selfish but not altruistic. Together with the second equation, we know that the incentive to free ride (selfishness) dominates regardless of how low the potential free riding benefit is, which is measured by the second winning prize ϵ, or how strong the desire to fight (morality) is, which is measured by the first winning prize w. The second Lemma coincides with the standard war of attrition. The discount factor does not change active allies' strategies at equilibrium. However, it matters to the firm's strategies as Equations (4.3) and (4.7) show. The third lemma shows that, comparing the equilibrium strategies in two states, the players on two sides act differently. Consumers driven by selfish motivations are more likely to leave the conflict when potential support from the alliance exists. Soon we will see that their quitting probabilities increase in the size of the demand side n. Meanwhile, the firm surrenders with slightly lower probability when facing two players' challenge if and only if the continuation values satisfy the condition $\alpha_3 > \alpha_1 + \alpha_2$. We can provide numerical solutions to the firm's quitting probabilities which shows that $q_{(n)}$ may fluctuate with the number of consumers n around $q^*_{(1)}$ depending on the values we give. Nevertheless it has a clear slowly increasing trend. The numerical solutions are available upon request.

4.4 A Generalised Case: n-against-one Game

Now we consider a game where the firm is a large polluting firm that serves many consumers in the market. The game of war of attrition between n consumers and one firm is as illustrated in Table 4.3. Note that we change some notations as follows. The actions $S(F_k)$ represent the case where player i surrenders and k consumers fight. The actions $F(F_{k-1})$ represent the joint actions of k active fighting consumers where player i is among them. In either case, the game will continue in State-k in the next period. We denote the winning prize to the firm to be $v_{(n)}$ indexed by the number of consumers n. It is, as before, assumed to be large enough to cover one period of fighting cost. Thus it ensures the existence of mixed strategy equilibria — the firm would not quit the conflict in the first period. The prize is tempting enough for the firm to stay in the costly war of attrition. It is natural to assume that the winning prize for the firm increases in n. In a large economy, a big firm that serves many consumers would generate more sales revenue. For simplicity, let $v_{(n)} = vn$.

As before, we assume that n players on the demand side play symmetric strategy.

Table 4.3: A game of war of attrition between n consumers and one firm

		Opponent F	Opponent S	
consumer i	$F(F_{n-1})$	$-c, -mn$	$w, 0$	$[1-p_{(n)}]^n$
	$F(F_{n-2})$	$-c, -m(n-1)$	$w, 0$	$[1-p_{(n)}]^{n-1}p_{(n)}$
	...	...	...	
	$F(F_{k-1})$	$-c, -mk$	$w, 0$	$[1-p_{(n)}]^k[p_{(n)}]^{n-k}$
	...	...	...	
	$F(F_0)$	$-c, -m$	$w, 0$	$[1-p_{(n)}]^1[p_{(n)}]^{n-1}$
	$S(F_{n-1})$	$0, -m(n-1)$	$\epsilon, 0$	$[p_{(n)}]^1[1-p_{(n)}]^{n-1}$
	...	...	...	
	$S(F_k)$	$0, -mk$	$\epsilon, 0$	$[p_{(n)}]^{n-k}[1-p_{(n)}]^k$
	...	...	...	
	$S(F_1)$	$0, -m$	$\epsilon, 0$	$[p_{(n)}]^{n-1}[1-p_{(n)}]$
	$S(F_0)$	$0, v$	$\epsilon, 0$	$[p_{(n)}]^n$
		$1-q_{(n)}$	$q_{(n)}$	

[a] The actions of the other players on the demand side in parentheses. F_{k-1} represents the case where among the other $n-1$ consumers on the demand side, there are $k-1$ fighters and $n-k$ quitters.
[b] The flow payoff to player i and the firm are in the cells. The payoffs to other allies, either active fighters or early quitters, are not shown in the table.
[c] The stopping probabilities are $(p_{(n)}, ..., p_{(n)}, q_{(n)})$ where n represents the number of the remaining players on the demand side. Identical consumers are assumed to play the same strategy if the state of the game does not change.
[d] The mixed strategy equilibrium is $(p^*_{(n)}, ..., p^*_{(n)}, q^*_{(n)})$ in State-n.
[e] In State-n at equilibrium, the continuation values to player i are $\alpha_{a_i,k}$ where a_i is player i's action, either fight F or surrender S. k is the number of active fighters excluding player i. There are $2n-1$ continuation values for player i. At equilibrium, the continuation values to the firm are always zero therefore not highlighted separately.

The quitting probability $p_{(n)}$ is the solution to the firm's indifference condition

$$
\begin{aligned}
& U_o(F) = U_o(S) \\
\Leftrightarrow \quad & \sum_{k=1}^{n}[-km]\binom{n}{k}[1-p_{(n)}]^k[p_{(n)}]^{n-k} + v_{(n)}[p_{(n)}]^n = 0 \\
\Leftrightarrow \quad & mnp_{(n)} + v_{(n)}[p_{(n)}]^n = mn && (4.8) \\
\Leftrightarrow \quad & mp_{(n)} + v[p_{(n)}]^n = m && (4.9)
\end{aligned}
$$

where $\binom{n}{k}$ is a binomial coefficient. k represents the number of active fighters. The continuation values βs are equal to 0 at equilibrium and therefore omitted. For any $n > 0$, $m > 0$ and $v_{(n)} > mn$, Equation (4.9) has a positive root in the range of $(0, 1)$, a negative root and $n-2$ imaginary roots if n is an even number. If instead n is odd, the equation has a positive root in the range of $(0, 1)$ and $n-1$ imaginary roots. The following results are derived.

Lemma 11. *Consumers' quitting probabilities $p_{(n)}$ increase in the size of the alliance n. consumers fight less hard when they have potential support from the alliance.*

Proof:

Rewriting the indifference condition Equation (4.9), we define the function $X(p_{(n)}) = mp_{(n)} + v[p_{(n)}]^n - m$. By the implicit function theorem

$$\partial p_{(n)}/\partial n = -\frac{\partial X(\cdot)/\partial n}{\partial X(\cdot)/\partial p_{(n)}} = -\frac{[p_{(n)}]^n v \ln p_{(n)}}{m + n[p_{(n)}]^{n-1} v} > 0 \qquad \square$$

Lemma 12. *In an n-against-one game where the firm fights against a side of n consumers, when the number of consumers goes to infinity, there is a unique solution to consumer's quitting probability $\lim_{n\to+\infty} p^*_{(n)}$ at equilibrium at the limit. The limit of $p^*_{(n)}$ is 1.*

Proof: First we prove that there exists a unique real solution $p_{(n)}$ s.t. $X(p_{(n)}) = 0$ for any given $m, n, v > 0$. It is clear that $X(p_{(n)})$ is continuous on the interval $[0, 1]$. $X(0) \to -m < 0$ when $p_{(n)} \to 0$ and $X(1) \to v > 0$ when $p_{(n)} \to 1$. By the Intermediate Value Theorem, there exists at least one $p \in (0, 1)$ s.t. $X(p) = 0$. Suppose there are two solutions p_1 and p_2 s.t. $p_1 \neq p_2$ and $X(p_1) = X(p_2) = 0$. By Rolle's theorem, this implies that $X'(p) = 0$. But $X'(p) = m + n[p_{(n)}]^{n-1}v > 0$ which is a contradiction.

Now we find the limit. We view $p_{(n)}$ as a sequence with $0 < p_{(n)} < 1$ satisfying Equation (4.9).

$$mp_{(n)} + v[p_{(n)}]^n = m$$

Divide by $p_{(n)}$ and let $y = v/m$,

$$LHS := 1 + y[p_{(n)}]^{n-1} = \frac{1}{p_{(n)}} =: RHS$$

Suppose that the limit of $p_{(n)}$ is not 1. Since $p_{(n)}$ increases in n, the limit, if there exists any, must be close to 1. Denote the limit as $\tilde{p} \pm \gamma$ where γ is some small number. We thus have $(\tilde{p} \pm \gamma)^{n-1} \to 0$ as $n \to \infty$. Plugging this into the LHS gives us

$1 + y[\widetilde{p} \pm \gamma]^{n-1} \to 1$ while the RHS goes above 1, which is a contradiction. Therefore the limit of an individual consumer's quitting probability is

$$\lim_{n \to +\infty} p_{(n)} = 1. \quad \square \tag{4.10}$$

Plugging the last equation into Equation (4.9), we immediately get the following results:

$$[p_{(n)}]^n \to 0 \tag{4.11}$$

$$[1 - p_{(n)}]^n \to 0 \tag{4.12}$$

The limits tell us how consumers' behave in a boycott. In the very first period, an individual consumer will almost surely surrender immediately driven by free riding incentives. However, not all of them will surrender. That is to say, the polluting firm will only face limited boycotting power from the consumers, but the boycott will not end soon, given the infinitely large number of consumers. Some will act as 'loyal supporters of the product' who would not boycott the firm, while the rest will act as 'strict environmentalists' who boycott for a considerably long period of time. The firm would thus expect to suffer some financial loss and damage to its corporate image from the boycott. However, if it is patient enough, such loss will reduce over time due to the decreasing number of active boycotters.

For the firm, the quitting probability $q_{(n)}$ is the solution to an individual consumer's indifference condition

$$\begin{aligned} U_i(F) = U_i(S), \quad i = 1, 2, ..., n \\ \Rightarrow wq_{(n)} + [1 - q_{(n)}] \sum_{k=0}^{n-1} (-c + \alpha_{F,k}\delta)[1 - p_{(n)}]^k \binom{n-1}{k} [p_{(n)}]^{n-1-k} \\ = \epsilon q_{(n)} + [1 - q_{(n)}] \sum_{k=1}^{n-1} \alpha_{S,k}\delta[1 - p_{(n)}]^k \binom{n-1}{k} [p_{(n)}]^{n-1-k} \end{aligned} \tag{4.13}$$

where k represents the number of fighters excluding player i. The continuation values $\alpha_{F,k}$ and $\alpha_{S,k}$ (indexed by k) are strictly positive and upper bounded at equilibrium. There are $2n - 1$ continuation values in State-n overall. When there are k active fighters remaining in the conflict,

$$\alpha_{F,k-1} = \alpha_{F,k-1}\Big(p_{(k)}, q_{(k)}\Big) = \epsilon q_{(k)} + \sum_{i=1}^{k-1} \alpha_{S,i}\delta\bigg[[1-p_{(k)}]^i\binom{k-1}{i}[p_{(k)}]^{k-1-i}[1-q_{(k)}]\bigg] \tag{4.14}$$

represents the continuation value that player i fights with $k-1$ remainers and

$$\begin{aligned}\alpha_{S,k} &= \alpha_{S,k}\Big(p_{(k)}, q_{(k)}\Big)\\ &= \frac{1}{1-[1-p_{(k)}]^k[1-q_{(k)}]\delta}\left\{\epsilon q_{(k)} + \sum_{i=1}^{k-1}\alpha_{S,i}\delta\bigg[[1-p_{(k)}]^i\binom{k}{i}[p_{(k)}]^{k-i}[1-q_{(k)}]\bigg]\right\}\end{aligned} \tag{4.15}$$

represents the continuation value that player i surrenders and k players remain in the conflict. In particular, when every player on the demand side surrenders, the game ends immediately and therefore there is no continuation value, that is,

$$\alpha_{S,0} := 0. \tag{4.16}$$

The following results are derived.

Lemma 13. *In an n-against-one game where the firm fights against a side of n consumers, when the number of consumers goes to infinity, there is a unique solution to the firm's quitting probability $\lim_{n\to+\infty} q^*_{(n)}$ at equilibrium at the limit.*[6] *The limit of $q^*_{(n)}$ is a function of c, w, ϵ and less than 1. That is, the firm tends not to surrender. It always plays mixed strategies.*

[6]The numerical solution for $q_{(n)}$ where $n = 1, 2, ..., 5$ given different ϵs is available upon request. It shows that $q_{(n)}$ fluctuate and increase slowly with n. It implies that the size of the firm has limited impact on its strategy making.

Proof: We rewrite the indifference condition (4.13), for $q_{(n)} \in (0,1)$ and $\delta \in (0,1)$,

$$
\begin{aligned}
& wq_{(n)} + [1-q_{(n)}]c + [1-q_{(n)}]\left\{\sum_{k=0}^{n-1} \alpha_{F,k}\delta[1-p_{(n)}]^k \binom{n-1}{k}[p_{(n)}]^{n-1-k}\right\} \\
&\quad = \epsilon q_{(n)} + [1-q_{(n)}]\sum_{k=0}^{n-1} \alpha_{S,k}\delta[1-p_{(n)}]^k \binom{n-1}{k}[p_{(n)}]^{n-1-k} \\
&\Rightarrow \alpha_{F,n-1}[1-p_{(n)}]^{n-1} + \sum_{k=0}^{n-2} \alpha_{F,k}[1-p_{(n)}]^k \binom{n-1}{k}[p_{(n)}]^{n-1-k} \\
&\quad - \sum_{k=1}^{n-1} \alpha_{S,k}[1-p_{(n)}]^k \binom{n-1}{k}[p_{(n)}]^{n-1-k} \\
&\quad = \frac{1}{[1-q_{(n)}]\delta}\Big\{c[1-q_{(n)}] - wq_{(n)} + \epsilon q_{(n)}\Big\}
\end{aligned} \tag{4.17}
$$

where the continuation values $\alpha_{F,k}$ and $\alpha_{S,k}, \forall k$ are upper bounded and certainly, for example, less than w which is the maximum utility an individual consumer can possibly get. The continuation value $\alpha_{F,n-1} = \alpha_{F,n-1}\Big(p_{(n)}, q_{(n)}\Big)$ is a function of $p_{(n)}$ and $q_{(n)}$. Therefore we write it separately. We take the limit on both sides.

$$
\begin{aligned}
\lim_{n\to+\infty} LHS &= \lim_{n\to+\infty}\Bigg\{\alpha_{F,n-1}[1-p_{(n)}]^{n-1} + \sum_{k=0}^{n-2} \alpha_{F,k}[1-p_{(n)}]^k \binom{n-1}{k}[p_{(n)}]^{n-1-k} \\
&\qquad - \sum_{k=1}^{n-1} \alpha_{S,k}[1-p_{(n)}]^k \binom{n-1}{k}[p_{(n)}]^{n-1-k}\Bigg\} \\
&= \lim_{n\to+\infty} \alpha_{F,n-1}[1-p_{(n)}]^{n-1} + \lim_{n\to+\infty} \sum_{k=0}^{n-2} \alpha_{F,k}[1-p_{(n)}]^k \binom{n-1}{k}[p_{(n)}]^{n-1-k} \\
&\qquad - \lim_{n\to+\infty} \sum_{k=1}^{n-1} \alpha_{S,k}[1-p_{(n)}]^k \binom{n-1}{k}[p_{(n)}]^{n-1-k}
\end{aligned} \tag{4.18}
$$

The first term of Equation (4.18) goes to zero at the limit since $\alpha_{F,n-1}$ is upper bounded and, $[1-p_{(n)}]^{n-1} \to 0$. Now we show that the second term and, by the same argument, the last term also approach zero. Let $G_{(k)} = [1-p_{(n)}]^k \binom{n-1}{k}[p_{(n)}]^{n-1-k}$. The greatest value $\bar{G}$, or called the mode, is given by $k = (n-1-1)p_{(n)} = np_{(n)}$ if k is an integer; otherwise, the greatest value is given by largest integer k such that $k \le np_{(n)}$.

$$
\bar{G} = (1-p_{(n)})^{np_{(n)}} p^{-1+n-np_{(n)}} \binom{n-1}{np_{(n)}} \tag{4.19}
$$

To find the limit of $q_{(n)}$, now we try to find an upper bound for $p_{(n)}$ which is lower than one. We consider this: $\forall j$ such that $n > j > 0$,

$$p_{(n)} \leq \widetilde{p} := 1 - \frac{j}{n} \text{ for large } n$$

This must hold.[7] [8] Then we have

$$p_{(n)}^n \leq \widetilde{p} := (1 - \frac{j}{n})^n \to e^{-j} > 0$$

Plugging $\widetilde{p}$ into Equation (4.19), we find that for any countable $n > 0$ and $p_{(n)}$ that satisfies Equations (4.10)-(4.12), when n increases to infinity,

$$\bar{G} = \frac{(1 - j/n)^j (j/n)^{-j+n} n \binom{n-1}{n-j}}{n - j} \to 0 \tag{4.20}$$

And,

$$\begin{aligned}
\sum_{k=0}^{n-2} \alpha_{F,k} [1 - p_{(n)}]^k \binom{n-1}{k} [p_{(n)}]^{n-1-k} &< \sum_{k=0}^{n-2} w (1 - p_{(n)})^{np_{(n)}} p^{-1+n-np_{(n)}} \binom{n-1}{np_{(n)}} \\
&= (n-1) w (1 - p_{(n)})^{np_{(n)}} p^{-1+n-np_{(n)}} \binom{n-1}{np_{(n)}} \\
&< (n-1) w \frac{(1 - j/n)^j (j/n)^{-j+n} n \binom{n-1}{n-j}}{n - j} \to 0
\end{aligned}$$

---

[7] We find a sequence $\widetilde{p}$ such that Equations (4.10) and (4.11) satisfy. Let $\forall n$ and j such that $n > j > 0$,

$$\widetilde{p} := 1 - \frac{j}{n}$$

$\forall k > 0, \widetilde{p} = 1 - \frac{j}{n}$ is monotonically increasing in n.

$$\lim_{n \to \infty} \widetilde{p} = 1 - \frac{j}{n} = 1$$

and

$$\widetilde{p}^n = (1 - \frac{j}{n})^n = \left[\left(1 - \frac{1}{\frac{n}{j}} \right)^{\frac{n}{j}} \right]^j = (e^{-1})^j = e^{-j} \in (0, 1)$$

As $n \to +\infty$,

$$[1 - (1 - \frac{j}{n})]^k \binom{n-1}{k} [1 - \frac{j}{n}]^{n-1-k} \to \frac{e^{-j} j^k}{\Gamma(1+k)} \in (0, 1)$$

and

$$\sum_{k=0}^{\infty} \frac{e^{-j} j^k}{\Gamma(1+k)} = 1$$

[8] The upper bound $\widetilde{p}$ is good enough to find the limit of $q_{(n)}$. We can prove, for example, that for $b < 1$, $p_{(n)} \leq \widetilde{p}_1 := 1 - b\frac{\ln n}{n}$ is a even lower upper bound. We can also show that for a small finite n, $p_{(n)} \leq \widetilde{p}_2 := 1 - \frac{b}{1+bn}$. The proofs are available upon request.

Analogously, the last term of Equation (4.17) approaches zero at the limit, which makes the left hand side a summation of three zeros.

The limit of the right hand side of Equation (4.17) is

$$\begin{aligned}
&\lim_{n\to+\infty} LHS = \lim_{n\to+\infty} RHS \\
\Leftrightarrow& 0 = \lim_{n\to+\infty} \frac{1}{[1-q_{(n)}]\delta}\Big\{c[1-q_{(n)}] - wq_{(n)} + \epsilon q_{(n)}\Big\} \\
\Leftrightarrow& 0 = \lim_{n\to+\infty} \frac{1}{[1-q_{(n)}]\delta} = 0 \qquad (4.21) \\
&\text{or} \lim_{n\to+\infty} c[1-q_{(n)}] - wq_{(n)} + \epsilon q_{(n)} = 0 \qquad (4.22)
\end{aligned}$$

Equality (4.21) is true if and only if $\lim_{n\to+\infty}[1-q_{(n)}]\delta \to +\infty$, that is, $\lim_{n\to+\infty} q_{(n)} \to -\infty$ which cannot be true. Therefore Equality (4.22) gives the unique solution for the limit of $q_{(n)}$.

$$\lim_{n\to+\infty} q_{(n)} = \frac{c}{c+w-\epsilon} < 1 \quad \square$$

Proposition 6. *In an n-against-one game where the firm fights against a side of n consumers, when the number of consumers goes to infinity, there exists a unique equilibrium* $(p^*_{(n)}, ..., p^*_{(n)}, q^*_{(n)})$ *in the game where identical players on the demand side are assumed to play symmetric strategies.*

4.5 Conclusion

We develop an uneven game of war of attrition between a weak alliance of two consumers and one firm. We examine the extension of free riding incentives and alliance advantage to Maynard Smith (1974)'s classic war of attrition under complete information. While two sides play a war of attrition, two consumers on one side play a prisoner's dilemma against each other. We allow the free rider to receive a prize if the game eventually ends with his ally's success. We derive the Nash equilibrium and compare the results to that of a classic one-against-one war of attrition. For the alliance, we find that selfish motivation overcomes moral concerns — free riding incentives dominate regardless of how strong the willingness to beat the firm is. Consumers fight less hard when they have more potential support from the alliance. The consumer's probability to surrender increases to one much more quickly when compared to the firm's equilibrium strategy.

We then extend the model to a general case where the size of the demand side increases. When the market size goes to infinitely large, we find that both sides quit the game with higher probabilities at equilibrium. Consumers would almost surely surrender immediately. Meanwhile, the firm's equilibrium strategies do not change significantly regardless of the number of the competitors. Therefore the game slowly becomes a one-sided war of attrition where the firm cannot be defeated easily by the consumers.

We employ the model to explain consumer boycott where consumers on the demand side fight against a misbehaving monopolist on the supply side. Our results suggest that in a market where a big firm serves many consumers, the real boycotting power is limited by consumers' free riding incentives. Over time the polluting firm suffers less costs due to the decreasing number of active boycotters. That is to say, for a strong enough firm that can survive the first several tough periods, the game will slowly favour the amoral firm and make it difficult for the consumers to win. Another possible real-life application of this model can be a labour strike where mass employees fight against an employer that offers unfair wages. The results therefore suggest the importance of third party intervention, either by the government or a non-profit organization. To make a boycott or a strike more effective and successful, a well-organized boycott is recommended.

Appendix

The detailed solution to the firm's quitting probabilities at equilibrium at State-2 in Equation (4.7) is

$$
\begin{aligned}
q^*_{(2)} = {} & (4\alpha_2\delta^2 m^2 + \delta(-\alpha_1 + \alpha_2(-1+8\delta) - \epsilon)mv - 4\alpha_2\delta^2 m\sqrt{m(m+2v)} + \alpha_1\delta v\sqrt{m(m+2v)} \\
& + \alpha_2\delta v\sqrt{m(m+2v)} - 4\alpha_2\delta^2 v\sqrt{m(m+2v)} + \delta\epsilon v\sqrt{m(m+2v)} - v^2(c - \epsilon + \delta(\alpha_2 - 2\alpha_2\delta + \epsilon) + w) \\
& + v^2\Big[1/v^4(-4\delta(m+v-\sqrt{m(m+2v)})(\epsilon v - \alpha_2\delta(m+v-\sqrt{m(m+2v)}))(cv^2 + \alpha_1\delta v(m - \sqrt{m(m+2v)}) \\
& + \alpha_2\delta(m+v-\sqrt{m(m+2v)})(v - \delta(m+v) + \delta\sqrt{m(m+2v)})) \\
& + (cv^2 + (-1+\delta)\epsilon v^2 + \delta(\alpha_1+\epsilon)v(m - \sqrt{m(m+2v)}) \\
& + \alpha_2\delta(m+v-\sqrt{m(m+2v)})(v - 2\delta(m+v) + 2\delta\sqrt{m(m+2v)}) + v^2 w)^2)\Big]^{1/2} / (2\delta(-m-v \\
& + \sqrt{m(m+2v)})(\epsilon v - \alpha_2\delta(m+v) + \alpha_2\delta\sqrt{m(m+2v)}))
\end{aligned}
$$

where α_1 and α_2 are as Equations (4.2) and (4.4) show. The solution is calculated by Mathematica.

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Chapter 5

A Note on Firms' Ethics, Consumer Boycotts, and Signalling[1]

5.1 Introduction

Glazer et al. (2010) develop a duopoly model of consumer boycott. The model discusses how a boycott affects firms' strategies in choosing production technology and output. We first present the model briefly and then discuss the determination of equilibrium strategies in Section 5.1. We derive the subgame perfect equilibria of the whole game in Section 5.2. In the rest of the article, we propose a solution to the model by considering Bertrand competition as an extension to the original article. In Section 5.3, we discuss the settings and key assumptions. In Sections 5.4 and 5.5, we derive the firms' equilibrium strategies by backward induction. Section 5.6 concludes the article.

In a market, two firms compete for market share by selling physically identical products. The products are however differentiable by the production process, either clean or polluting. With an additional fixed production cost c, a firm can choose the expensive clean technology. Producing with the polluting technology does not require any additional cost. Consumers view the clean firm as ethical (H) and the polluting firm as non-ethical (L).

There are two types of consumers: a-type is high moral and b-type is low moral. The numbers of a-type and b-type are scaled to n and 1, respectively. Consumers are utility maximising taking account of the cost of social pressure b. The cost arises only

[1]This chapter is an extension to a published article written by Glazer et al. (2010).

from non-boycotting behaviour, that is, buying from the polluting firm and ignoring the firm's ethical code. Buying from the clean firm does not involve such cost. Consumers of a-type choose between buying at Firm H or not buying anything. Based on their willingness to pay for the products, a-type consumers are indexed in decreasing order on $i \in [0, n]$. The utility function of a-type is given by

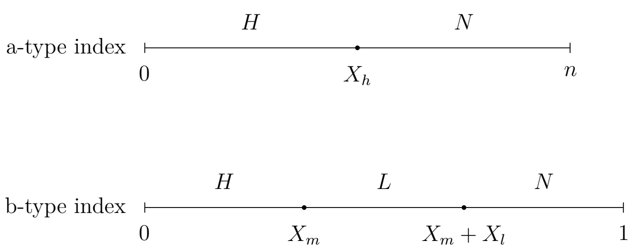
$$U_i = \begin{cases} \beta(1 - i/n) - P_H, & \text{if she/he chooses Firm } H \\ 0, & \text{otherwise} \end{cases} \tag{5.1}$$

where P_H is the asking price set by Firm H and β is some scale to the utility gain from purchasing. A larger β indicates higher valuation for the product. Consumers of b-type can choose from buying at either firm or buying nothing. Based on their willingness to pay for the products, b-type consumers are indexed in decreasing order on $j \in [0, 1]$. The utility function of b-type is given by

$$U_j = \begin{cases} \beta(1 - j) - P_H, & \text{if she/he chooses Firm } H \\ \beta(1 - j) - P_L - b, & \text{if she/he chooses Firm } L \\ 0, & \text{otherwise} \end{cases} \tag{5.2}$$

where P_L is the product price set by Firm L and b is the cost of social pressure if a consumer buys from the polluting firm.

Figure 5.1: Consumers' Willingness to Pay and Choices



Note: The willingness to pay of a-type consumers is in $[0, n]$ in decreasing order. The willingness to pay of b-type consumers is in $[0, 1]$ in decreasing order. The choice variables $a_j \in \{H, L, N\}$ represent the actions of purchasing from Firm H, purchasing from Firm L, and not buying anything.

Figure 5.1 illustrates the consumers' willingness to pay and the preferred actions for both types. For a-type consumers, as 0 (at the left end of the first line) represents the highest willingness to pay and their willingness decreases when moving towards n at the right end, there must exist some marginal consumer X_h in the middle of the line who is indifferent between Firm H and not buying. Similarly, for b-type consumers, since they have three choices, there must exist two marginal consumers such that in the range of $[0, X_m)$, they strictly prefer Firm H; in the range of $(X_m, X_m + X_l)$, they strictly prefer Firm L; otherwise they buy nothing.

The critical marginal consumers are defined by the authors of the original article as follows. Note that we have changed the notation J to avoid confusion.

> X_m: *"The benefit from signalling is sufficient so that some, i.e., X_m, do. The marginal low-moral consumer must be indifferent between the two markets."*
> $J = X_m + X_l$: *"The marginal low-moral consumer (with an index $J = X_m + X_l$) is indifferent between buying at Firm L or buying nothing."*

The authors thus solve the indifference condition for the marginal consumers and find that

> *"all the low-moral types are indifferent between the two markets."*

The equilibrium is derived under the condition that the price difference $P_H - P_L$ is equal to the cost of social pressure b (Lemma 1 of Glazer et al. (2010)). The optimal outputs at equilibrium are given by

$$\begin{cases} Y_H = X_h + X_m \\ Y_L = X_l \end{cases}$$

where $X_m = \frac{1-n}{3} + \frac{(1+2n)b}{3\beta}$ as solved in Corollary 1. X_m is unique since n, β and b are exogenously given.

5.1.1 Marginal Consumer and Equilibrium Determination

Now we go back to the definition of X_m and Lemma 1 of Glazer et al. (2010). We find that the consumer indexed by X_m is not uniquely determined. Since all b-type consumers are indifferent between two firms, they all fit the criteria of being the marginal. If the benefit from buying at Firm H is sufficient for some amount of consumers X_m,[2]

[2] X_m has two meanings. As stated in Section Assumptions on Page 341, consumers are indexed in decreasing order on [0,1] therefore the $m-th$ consumer with willingness to pay X_m is the marginal. X_m also represents the total number of b-type purchasing consumers at Firm H.

such benefit should be also sufficient for every b-type consumer. These make each b-type a potential marginal consumer.

Instead of having one marginal consumer X_m, we have many and they are indexed in the range of $[0, X_m + X_l]$. The rest indexed in $(X_m + X_l, 1]$ maximize their utilities by not buying anything.

$$\max_{a_j \in \{H,L,N\}} \{U_j(H), U_j(L), U_j(N)\} = 0 =: U_j(N)$$

where $U_j(H) = U_j(L)$. The marginal consumer $J = X_m + X_l$ is indifferent from buying at either firm and buying nothing, where the value of J depends on X_m.

Determining the marginal consumers X_m and J is critical since it affects the determination of firms' equilibrium outputs. Both X_m and J have double meanings. They also tell us how many consumers would buy from each firm. For a pair of marginal consumers (X_m, J), we would find a unique solution to the equilibrium outputs. This means that we shall have a continuous set of equilibria given that the marginal consumers are not uniquely determined.

At equilibrium, the output of H firm, Y_H, can vary from X_h, if it only serves a-type, to $X_h + X_m + X_l$, if it serves both types. Meanwhile L firm's output Y_L is $X_m + X_l$ at maximum and 0 at minimum, depending on the assumption imposed on the preferred moves of the indifferent consumers. In the worst case scenario, L firm can be 'forced' to leave the market since it cannot make trades with any consumers.

We construct equilibria in two polar cases, just to demonstrate our point. First, since all b-type consumers in $[0, X_m + X_l]$ are indifferent between two firms, let us assume they all pick H Firm. Firms' outputs are thus

$$\begin{cases} Y_H = X_h + X_m + X_l \\ Y_L = 0. \end{cases} \tag{5.3}$$

Plug it into the condition in Lemma 1 of Glazer et al. (2010),

$$\begin{aligned} P_H - P_L &= b \\ \beta(1 - X_h/n) - [\beta(1 - X_m - X_l) - b] &= b \\ \Rightarrow X_m + X_l = \frac{Y_H}{n+1} &= X_h/n. \end{aligned}$$

Firm H maximizes its profit by choosing the quantity Y_H.

$$\begin{aligned} \max_{Y_H} \pi_H &= Y_H P_H - c \\ &= Y_H \beta(1 - X_h/n) - c \\ &= Y_H \beta(1 - \frac{Y_H}{n+1}) - c, \end{aligned}$$

where c is the fixed cost of production. The first order condition with respect to Y_H gives

$$\begin{aligned} \beta - \frac{2\beta}{n+1} Y_H &= 0 \\ \Rightarrow Y_H &= \frac{n+1}{2}. \end{aligned} \tag{5.4}$$

Therefore the equilibrium outputs are given by Equations (5.3) and (5.4). Recall that the numbers of a-type and b-type consumers are n and 1, respectively. The last equation tells us that the ethical Firm H would serve half of the population in the market. Meanwhile the non-ethical Firm L would find it difficult to sell anything.

Second, we assume that all the indifferent consumers in $[0, X_m + X_l]$ choose Firm L. Firms' outputs are thus

$$\begin{cases} Y_H = X_h \\ Y_L = X_m + X_l. \end{cases} \tag{5.5}$$

Plugging these to the condition $P_H - P_L = b$, we get

$$\begin{aligned} \beta(1 - X_h/n) - [\beta(1 - X_m - X_l) - b] &= b \\ \Rightarrow X_m + X_l = X_h/n &= Y_H/n. \end{aligned}$$

Firm H maximizes its profit by choosing the quantity Y_H.

$$\begin{aligned}\max_{Y_H} \pi_H &= Y_H P_H - c \\ &= Y_H \beta(1 - X_h/n) - c \\ &= Y_H \beta(1 - Y_H/n) - c.\end{aligned}$$

The first order condition with respect to Y_H gives

$$\begin{aligned}&\beta - \frac{2\beta}{n} Y_H = 0 \\ &\Rightarrow Y_H = \frac{n}{2} \qquad (5.6) \\ &\Rightarrow Y_L = X_m + X_l = Y_H/n = \frac{1}{2}. \qquad (5.7)\end{aligned}$$

Therefore the equilibrium outputs are given by Equations (5.6) and (5.7).

To summarize, there should exist infinitely many equilibria and the equilibrium determined by the authors of the original article is one of them. However, the equilibrium can be unique by, as shown above, assuming that all the indifferent consumers choose Firm L (or H), that is, they all prefer to (or prefer not to) endure the social pressure and pay the additional cost b. Alternatively, we can assume that the indifferent consumers split into two groups according to some criteria. For example, half of them buy from Firm L and the other half buy from Firm H.

5.2 Subgame Perfect Equilibria

We have discussed the equilibrium uniqueness and constructed equilibria in two polar cases following the method used in the original article. In this section, we solve the subgame perfect equilibria.

5.2.1 Optimal Outputs

The game is solved backwards. In the second stage, two firms decide the outputs. We introduce a new variable: denote the number of b-type low-moral consumers who purchase from Firm H by $x \in [0, J]$. When $x = 0$, the b-type consumers who are indifferent between two firms choose Firm L. When $x = J$, the indifferent consumers choose Firm H. Thus two firms' outputs are

$$\begin{cases} Y_H = X_h + x \\ Y_L = X_m + X_l - x = J - x. \end{cases} \tag{5.8}$$

Firm H maximizes its profit by choosing the optimal output with the fixed cost of production c.

$$\begin{aligned} \max_{Y_H} \pi_H &= Y_H P_H - c \\ &= Y_H \beta(1 - X_h/n) - c \\ &= Y_H \beta(1 - Y_H/n + x/n) - c. \end{aligned}$$

The first order condition with respect to Y_H gives

$$Y_H = \frac{n + x}{2}. \tag{5.9}$$

Meanwhile, Firm L receives profit without additional cost of production.

$$\begin{aligned} \max_{Y_L} \pi_L &= Y_L P_L \\ &= Y_L \beta(1 - J) \\ &= Y_L \beta(1 - Y_L - x). \end{aligned}$$

The first order condition with respect to Y_L gives

$$\begin{aligned} \beta - 2\beta Y_L - \beta x = 0 \\ \Rightarrow Y_L = \frac{1 - x}{2}. \end{aligned} \tag{5.10}$$

Plugging the optimal outputs into the profit functions, we get the equilibrium profits when two firms choose different production technology.

$$\begin{cases} \pi_H = \frac{-4cn + b(n+x)^2}{4n} \\ \pi_L = \frac{1}{4} b(x - 1)^2. \end{cases} \tag{5.11}$$

A firm has an incentives to adopt the clean technology only if the fixed cost c is sufficiently low, that is

$$\begin{aligned} &\pi_H \geq 0 \\ &\Rightarrow c \leq \frac{bn^2 + 2bnx + bx^2}{4n} \end{aligned} \tag{5.12}$$

Since the value of x varies depends on the assumption imposed on the preference of indifferent consumers, as discussed above, there are infinitely many equilibria. However, for each x, the equilibrium is unique.

The determination of the marginals does not affect the solutions to the optimal outputs when two firms choose the same technology. Therefore we do not repeat the calculation here. As shown in the original article, when both firms choose the polluting technology L, they are indexed by A and B. The optimal outputs are

$$Y_f^{LL} = \frac{\beta - b}{3\beta}, f \in \{A, B\}$$

and the profits are

$$\pi_f^{LL} = \frac{(\beta - b)^2}{9\beta}.$$

When both firms choose the clean technology H, the optimal outputs are

$$Y_f^{HH} = \frac{1+n}{3}, f \in \{A, B\}$$

and the profits are

$$\pi_f^{HH} = \frac{(1+n)\beta}{9} - c_f.$$

5.2.2 Selection in Production Technology

Now we solve the strategies in technology selection in the first stage and therefore derive the subgame perfect equilibria of the game. In the first stage, two firms simultaneously choose the production technology, either clean (H) or polluting (L). The game is illustrated in Table (5.1). where c_f is the cost of investing in the clean technology for

Table 5.1: Payoff Matrix

		Firm B: H	Firm B: L	
Firm A	H	$\frac{(1+n)\beta}{9} - c_A$, $\frac{(1+n)\beta}{9} - c_B$	$\frac{-4c_A n+b(n+x)^2}{4n}$, $\frac{1}{4}b(x-1)^2$	q_A
	L	$\frac{1}{4}b(x-1)^2$, $\frac{-4c_B n+b(n+x)^2}{4n}$	$\frac{(\beta-b)^2}{9\beta}$, $\frac{(\beta-b)^2}{9\beta}$	$1-q_A$
		q_B	$1-q_B$	

[a] Firm $f, f \in \{A, B\}$ chooses from the clean technology H or the polluting technology L. The adoption of the clean technology needs the fixed production cost c_f.
[b] The probability $q_f, f \in \{A, B\}$ is the probability that firm f chooses the clean technology H.

each firm, $f \in \{A, B\}$, $q_f \in [0, 1]$ is the probability that firm f chooses to invest and x is the number of b-type indifferent consumers who choose firm H.

Depending on the assumption on consumers' preferences, the value of x can be uniquely pinned down. Since the number of a-type high-moral consumers n, the cost of social pressure b, the fixed production cost of clean technology c_f and the value of β are exogenously given, we can determine the equilibrium. Here we give an example.

Proposition 7. *There exists two pure strategy equilibria such that both firms choose the same technology, that is, (H, H) and (L, L) if the conditions*

$$\frac{(1+n)\beta}{9} - c_f \geq \frac{1}{4}b(x-1)^2$$
$$\frac{-4c_f n + b(n+x)^2}{4n} < \frac{(\beta-b)^2}{9\beta}$$

hold, where $f \in \{A, B\}$. There exists a mixed strategy equilibrium where firm f chooses the clean technology with the probability q_f that is given by

$$q_f = \frac{4b^2 n + 4\beta(\beta + 9c_{-f})n - b\beta[n(8+9n) + 18nx + 9x^2]}{4b^2 n + 4\beta^2 n(2+n) - b\beta[n(17+9n) + 9(1+n)x^2]}.$$

In the rest of the article, we propose a solution to the model by considering Bertrand competition as an extension to Glazer et al. (2010). In the settings of the model, three major changes to the original article are highlighted. The game is solved backwards. We first determine the firm's optimal pricing strategies in the second stage. Then we derive the firms' strategies in technology selection in the first stage.

5.3 Bertrand Competition

We employ a consumer boycott model and examine the role of boycotts in market competition. We discuss how consumers' moral values and boycotting behaviours affect firms' decision-making in pricing and production technology selection. We find out under what conditions firms are able to achieve positive profits. We study whether investing in the clean technology (behaving ethically) generates higher returns than using the polluting technology (behaving unethically). We keep the assumptions and settings of the original article unless stated otherwise.

In the duopoly market, two firms sell one type of product. The production technology can differ, either clean or polluting. There are two types of consumers in the market. Consumers of a-type are environmentalists and never buy from the polluting firm. The action set of a-type is binary $a_j \in \{H, N\}$ which represents the purchasing behaviour at Firm H or buying nothing, respectively. The mass of a-type consumers is n and they are indexed on $i \in [0, n]$. Consumers of b-type do not have environmental concerns so they choose the firm which brings them higher utility. The action set of b-type is $a_j \in \{H, L, N\}$ where L represents the purchasing behaviour from Firm L. The mass of b-type is 1 and they are indexed on $j \in [0, 1]$. In Glazer et al. (2010) model, β, as shown in Equations (5.1) and (5.2), is an exogenously given parameter in consumers' utility functions, i.e., its value is same to everyone. For simplicity, we set $\beta = 1$.[3] Thus the utility function of a-type consumer i becomes:

$$U_i = \begin{cases} 1 - \dfrac{i}{n} - P_H, & \text{if she/he chooses Firm } H \\ 0, & \text{otherwise} \end{cases} \tag{5.13}$$

The utility function of b-type consumer j becomes:

$$U_j = \begin{cases} 1 - j - P_H, & \text{if she/he chooses Firm } H \\ 1 - j - P_L - b, & \text{if she/he chooses Firm } L \\ 0, & \text{otherwise} \end{cases} \tag{5.14}$$

where the cost of social pressure $b > 0$ is common knowledge and sufficiently large so

[3]It can be shown that the selection of β does not change the main results.

that all a-type and some b-type consumers have strong incentive to boycott.[4] Since a-type consumers always boycott and b-type are potential boycotters, the size of a-type is very important to firms' technology selection. We assume the size n to be large enough so that a-type consumers have some power to potentially "force" firm(s) to perform environmentally-friendly actions. If this assumption is not satisfied, firm(s) can simply ignore the consumers' boycott since the potential financial loss arising from the boycott is very limited. We also assume that the cost of investing in the clean technology c is small enough so that firm(s) can afford it and is(are) willing to pay for it. Otherwise firm(s) would prefer the polluting technology with zero extra cost. We further assume that the product is a normal good but not a necessity. Otherwise consumers have less incentive to boycott. This explains why a-type consumers are able to not buy anything.

The game is played in two stages under complete information. In the first stage, firms simultaneously decide which production technology to adopt. In the second stage, firms choose the price. The game is solved backwards. Starting from the second stage, we consider the production technology that has been chosen in the first stage: (1) when one firm has invested in the clean technology; (2) when both firms have invested in the clean technology; and (3) when both firms have chosen the polluting technology. Then we move to the first stage and solve the strategies in technology selection. The subgame perfect equilibria are thus derived.

Three main changes are made as follows. First, we reform the model by permitting the relaxation of Lemma 1 ($P_H - P_L = b$). Firm H chooses the best response to Firm L's pricing strategy, which leads us to three possibilities: the price P_H is equal to, larger than, or smaller than the summation of the exogenously given cost b and the endogenously chosen P_L. Second, we make an assumption on indifferent consumers.

Assumption 2 (Sufficient social cost)**.** *For a large enough cost of social pressure b, the consumer who is indifferent between two firms buys from Firm H.*

Low-moral consumers favour Firm L if only if it brings strictly higher utility. However, for a small b, we impose a different assumption and provide the intuition later. Third, firms decide price. We contribute to the original work by deriving the equilibrium strategies in Bertrand competition settings.

[4] Later in this article, we discuss the special case when b is very small (see Assumption 3).

5.4 Pricing Strategies

The game is solved by backward induction. We first derive firms' optimal pricing strategies in the second stage conditional on firms' technology selection in the first stage. In each of the following cases, we derive the pricing strategies and discuss the role of the cost of social pressure b.

5.4.1 Case 1: Only One Firm Invests in the Clean Technology

We start from the case where only one of the two firms has invested in the clean technology in the first stage. Under the conditions that P_H is equal to, smaller than or larger than $P_L + b$, we check whether equilibria can exist in the the second stage. If there exists any, we derive the optimal pricing strategies. We show that there is no equilibrium under the condition of the cost of social pressure $b \geq P_H - P_L$; equilibria exist under the condition $b < P_H - P_L$.

5.4.1.1 $P_H = P_L + b$

Consider Firm H sets a considerably high price such that, to each consumer, the cost of purchasing from it (P_H) equals the total cost of purchasing at its rival ($P_L + b$). Consumers have different preferences and this is known to both firms. Consumers of a-type do not buy from Firm L. Thus Firm L's targeted consumer group is b-type. To each consumer j of b-type, buying from Firm H brings her/him the same utility as from Firm L.

$$\begin{aligned} U_j(H) &:= 1 - j - P_H \\ &= 1 - j - P_L - b =: U_j(L) \end{aligned}$$

By assumption, when b is sufficiently large, the consumer who is indifferent between two firms chooses Firm H. Hence no one in the market chooses Firm L. Firm H is able to choose any price level P_H as long as $P_H = P_L + b$ holds. The total mass of potential customers for Firm H is $1 + n$, i.e., Firm H may serve the entire market. The marginal a-type consumer indexed by X_h is indifferent between buying from Firm H and buying nothing:

$$\begin{aligned}U_i(H) &:= 1 - \frac{X_h}{n+1} - P_H \\ &= 0 =: U_i(N) \\ &\Rightarrow X_h = (1+n)(1-P_H)\end{aligned}$$

Firm H chooses the optimal price P_H^* to maximise its own profit π_H^{HL}:

$$\begin{aligned}&\max_{P_H} \pi_H^{HL} = X_h P_H - c \\ &\frac{\partial \pi_H^{HL}}{\partial P_H} = (n+1)P_H(-1) + (n+1)(1-P_H) = 0 \\ &\Rightarrow P_H^* = \frac{1}{2} \qquad (5.15)\end{aligned}$$

Meanwhile, Firm L sets $P_L = P_H - b = \frac{1}{2} - b$ and sells nothing. It is clearly not an equilibrium since Firm L can easily find profitable deviation by lowering the price P_L slightly to attract some b-type consumers.

5.4.1.2 $P_H < P_L + b$

Now consider that Firm H sets the price P_H to a low level that is less than the total cost of purchasing from its rival ($P_L + b$). Similar to the above case, Firm H attracts both a-type and b-type consumers. To each b-type consumer j, buying from Firm H brings her/him strictly higher utility than buying from Firm L.

$$\begin{aligned}U_j(H) &:= 1 - j - P_H \\ &> 1 - j - P_L - b =: U_j(L)\end{aligned}$$

No one in the market buys from Firm L since it is so costly to do so. Again Firm H is able to choose any price P_H as long as $P_H < P_L + b$ holds. The total mass of potential customers for Firm H is $1 + n$. It is optimal to set $P_H^* = \frac{1}{2}$. This is not an equilibrium since Firm L can do better by lowering the price P_L to attract b-type consumers.

5.4.1.3 $P_H > P_L + b$

For an equilibrium to be possible, the total cost from choosing Firm L ($P_L + b$) needs to be smaller than from choosing Firm H. That is, to b-type consumer j, buying from Firm L brings her/him strictly higher utility.

$$\begin{aligned} U_j(H) &:= 1 - j - P_H \\ &< 1 - j - P_L - b =: U_j(L) \end{aligned}$$

We check two conditions: (a) $b \geq \frac{1}{2}$; and (b) $b \in (0, \frac{1}{2})$. Under Condition (a), the cost of social pressure is high enough so that it is very hard for Firm L to get consumers. At the same time, Firm H enjoys the benefit of social pressure. The mass of potential consumers for Firm H is n. The marginal moral consumer X_h of a-type satisfies:

$$\begin{aligned} U_i(H) &:= 1 - \frac{X_h}{n} - P_H \\ &= 0 := U_i(N) \\ &\Rightarrow X_h = n(1 - P_H) \end{aligned}$$

Firm H chooses the optimal price P_H^* to maximize its own profit π_H^{HL}:

$$\begin{aligned} &\max_{P_H} \pi_H^{HL} = X_h P_H - c \\ &\frac{\partial \pi_H^{HL}}{\partial P_H} = (n+1)P_H(-1) + (n+1)(1 - P_H) = 0 \\ &\Rightarrow P_H^* = \frac{1}{2} \end{aligned}$$

Thus we find an equilibrium where the prices are $P_H = \frac{1}{2}$ and $P_L = 0$. Plugging the prices into the profit functions, we find that Firm H's profit is $\pi_H^{HL}(P_H) = \frac{n+1}{4} - c$ and Firm L's payoff is $\pi_L^{HL}(P_L) = 0$. Both have no incentive to deviate. Investing in the clean technology is profitable for large enough n and small enough c.

Proposition 8. *When two firms have chosen different technologies in the first stage of the game, for $b \in [\frac{1}{2}, +\infty)$, there is a unique equilibrium where Firm H with the clean technology sets the price $P_H = \frac{1}{2}$ and Firm L with the polluting technology sets the price*

$P_L = 0$ such that the profits are

$$\pi_H^{HL} = \frac{n+1}{4} - c$$

to Firm H and

$$\pi_L^{HL} = 0$$

to Firm L. *[EQU1]*

Under Condition (b) $b \in (0, \frac{1}{2})$, we consider the following candidate equilibria: (i) For a larger b, Firm H sets a relatively lower price ($P_H = b$) to attract both types; meanwhile Firm L chooses $P_L = 0$; and (ii) for a smaller b, Firm H increases the price ($P_H = \frac{1}{2}$) such that it gets a-type consumers and Firm L gets b-type; Firm L chooses $P_L = \frac{1}{2} - b > 0$.

We first derive the condition for (i) to be favoured over (ii). When $P_H = b$, since no one buys from Firm L, the mass of potential consumers for Firm H is $n + 1$. When $P_H = \frac{1}{2}$, the mass of potential consumers for Firm H decreases to n. We further assume that if Firm H is indifferent from choosing $P_H = b < 1/2$ and $P_H = \frac{1}{2}$, it prefers the former so that it can serve a bigger share of the market. Thus Firm H chooses (i) if it brings higher profit.

$$\begin{aligned}
\pi_H^{HL}(P_H = b) &\geq \pi_H^{HL}(P_H = \frac{1}{2}) \\
[(1+n)(1-b)]b - c &\geq [n(1-\frac{1}{2})]\frac{1}{2} - c \\
(1+n)(1-b)b &\geq \frac{n}{4} \\
&\Rightarrow \quad b^2(4+4n) + b(-4-4n) + n \leq 0 \\
&\Rightarrow \quad b \in [\frac{1}{2} - \frac{1}{2\sqrt{n+1}}, \frac{1}{2}) \qquad (5.16)
\end{aligned}$$

Firm H has no incentive to deviate from $P_H = b$ since it is derived by profit maximising. Also Firm L has no incentive to deviate from $P_L = 0$ since there is no way to generate non-zero profit when the conditions (5.16) and $P_H = b$ hold. So this is an equilibrium.

Proposition 9. *When two firms have chosen different technologies in the first stage of the game, for $b \in [\frac{1}{2} - \frac{1}{2\sqrt{n+1}}, \frac{1}{2})$, there is a unique equilibrium where Firm H with the clean technology sets the price $P_H = b$ and Firm L with the polluting technology sets* *the price $P_L = 0$ such that the profits are $\pi_H^{HL} = (1+n)(1-b)b - c$ to Firm H and $\pi_L^{HL} = 0$ to Firm L.* *[EQU2]*

Now we consider the candidate equilibrium (ii) $P_H = \frac{1}{2}$ and $P_L = \frac{1}{2} - b$. By $0 < P_L < P_H - b$, Firm L is able to attract b-type consumers and meanwhile Firm H only attracts a-type. Now each firm sets the price targeting only one type of consumer. Unlike the standard Bertrand duopoly model, it seems possible for both firms to obtain non-zero revenue.

When $b \in (0, \frac{1}{2} - \frac{1}{2\sqrt{n+1}})$, Firm H obtains the maximum profit by setting $P_H = \frac{1}{2}$. Firm L attracts some consumers as long as $P_L < P_H - b$ holds. Clearly at $P_H = \frac{1}{2}$, Firm L's profit $\pi_L^{HL}(P_L)$ is monotonically increasing in price P_L on the support $[0, \frac{1}{2} - b)$. Thus for a small positive number ϵ, Firm L obtains non-zero profit by setting $P_L = P_H - b - \epsilon = \frac{1}{2} - b - \epsilon$. In this special case, for an equilibrium to be possible, we make an assumption:

Assumption 3 (Insufficient social cost)**.** *For a small enough $b \in (0, \frac{1}{2} - \frac{1}{2\sqrt{n+1}})$, the consumer who is indifferent between two firms chooses Firm L.*

The interpretation is given as follows: When the cost of social pressure is not sufficiently large, society only considers firms' non-ethical behaviour (polluting) and consumers' amoral behaviour (not boycotting) as small issues. The society does not harshly censure non-ethical attitudes or behaviour. For low-moral (b-type) consumers, the benefit from boycotting (to avoid the social cost) is no longer significant for utility improvement. That is to say, for the indifferent consumers (who are indifferent between two firms), the incentives to engage in boycotting activity decrease with the cost of social pressure b. Thus we suggest that there is a cutoff point $\widehat{b} = \frac{1}{2} - \frac{1}{2\sqrt{n+1}}$ such that for any $b \geq \widehat{b}$, the indifferent consumer prefers to boycott the polluting firm(s); for any $b < \widehat{b}$, the indifferent consumer prefers to pay a lower product price (P_L) and not to boycott.

Under this assumption, we are now able to derive the pricing strategies for both firms. For a small enough b, there is an equilibrium where both firms obtain non-zero revenue by setting $P_H = \frac{1}{2}$ and $P_L = \frac{1}{2} - b$ respectively. Plugging the prices into the profit functions, we find that Firm H's profit is $\pi_H^{HL} = \frac{n}{4} - c$ and Firm L's profit is $\pi_L^{HL} = \frac{1}{4} - \frac{1}{2}b$.

Proposition 10. *When two firms have chosen different technologies in the first stage of the game, for $b \in (0, \frac{1}{2} - \frac{1}{2\sqrt{n+1}})$, there is a unique equilibrium where Firm H with the clean technology sets the price $P_H = \frac{1}{2}$ and Firm L with the polluting technology sets the price $P_L = \frac{1}{2} - b$ such that the profits $\pi_H^{HL} = \frac{n}{4} - c$ to Firm H and $\pi_L^{HL} = \frac{1}{4} - \frac{1}{2}b$ to Firm L.* *[EQU3]*

From the above results, we note that for a large enough n and a small enough c as assumed, Firm H is able to make non-zero profit at equilibria. The conditions are $c < \frac{n+1}{4}$ in *[EQU1]*; and $c < (1+n)(1-b)b$ in *[EQU2]*. Both firms are able to make non-zero profit at equilibrium if b is small enough: $c < \frac{n}{4}$ and $b < \frac{1}{2} - \frac{1}{2\sqrt{n+1}}$ in *[EQU3]*. This means that between two firms with different ethical codes, it is certainly beneficial to be social responsible and show concern for the environment (as Firm H does), but we cannot conclude that there is no benefit to be the non-ethical one (as Firm L is). In a society where the average level of social responsibility concern is sufficiently low, an unethical firm can make positive profits in price competition, and in some circumstances, even make higher profits than the high-ethics firm.

5.4.2 Case 2: Both Firms Invest in the Clean Technology

Now we consider the case where both firms have invested in the clean technology in the first stage. Let them be Firm A and Firm B. Each Firm needs to pay a fixed cost to adopt the clean technology $c_f, f \in \{A, B\}$ and there is no marginal cost. When two firms choose the same technology, either clean or polluting, it is similar to the standard Bertrand duopoly model: each firm sells perfectly identical products; two firms equally share the market by setting the same price; both firms make zero revenue.

When both firms use the clean technology, the total mass of the potential consumer is $1+n$. Let X_{HH} be the marginal consumer who is indifferent between buying and not buying, and P_H be the product price.

$$1 - \frac{X_{HH}}{1+n} - P_H = 0$$
$$\Rightarrow X_{HH} = (1+n)(1-P_H)$$

Let X_A and X_B be the number of consumers who choose Firm A and Firm B respectively.

$$X_A = X_B = \frac{1}{2}X_{HH} = \frac{1}{2}(1+n)(1-P_H)$$

Firm f makes profit $\pi_f^{HH}, f \in \{A, B\}$

$$\pi_f^{HH} = X_f P_H - c_f$$

Since the firm with the lower price attracts all the consumers, there is a unique equilibrium when both firms choose $P_H = 0$. Since they need to pay the cost of investing in the clean technology $c_f > 0, f \in \{A, B\}$, they actually make negative profits. The firm with the higher investing cost suffers a bigger loss. Two firms together serve the entire market, i.e., the mass of purchasing consumers is $1 + n$.

So, when there is no outside option such that firms can exit the market and obtain zero profit, for example, due to legal restrictions or the cost of exit, both firms choose the clean technology with zero product price.

Proposition 11. *When both firms have chosen the clean technology in the first stage of the game, $\forall b$, there is a unique equilibrium where both firms choose $P_H = 0$ such that the profits are $\pi_A^{HH} = -c_A$ and $\pi_B^{HH} = -c_B$.* *[EQU4]*

5.4.3 Case 3: No Firm Invests in the Clean Technology

Now we consider the case where no firm has invested in the clean technology in the first stage. The mass of potential consumers now reduces to 1 since a-type consumers do not buy from the polluting firms. Let X_{LL} be the marginal consumer who is indifferent between buying and not buying.

$$1 - X_{LL} - b - P_L = 0$$
$$\Rightarrow X_A = X_B = \frac{1}{2}X_{LL} = \frac{1 - b - P_L}{2}$$

where two firms equally share the market by setting the same price P_L.

Analogously to the case above, there is an equilibrium when $P_L = 0$ such that both firms obtain zero revenue. We notice that the mass of purchasing consumers is $1 - b$

and all of them are of b-type. That is, in such case two firms jointly serve the smallest share of the market.

Proposition 12. *When both firms have chosen the polluting technology in the first stage of the game, $\forall b$, there is a unique equilibrium where both firms choose $P_L = 0$ such that the profits are $\pi_A^{HH} = \pi_B^{HH} = 0$.* *[EQU5]*

5.5 Selection in Production Technology

We have derived the pricing strategies in the second stage conditional on firms' technology adopted in the previous stage. Now we solve the strategies in technology selection and therefore derive the subgame perfect equilibria of the game. In the first stage, two firms simultaneously choose which technology to use. We consider three cases: (a) $b \in [\frac{1}{2}, +\infty)$; (b) $b \in [\frac{1}{2} - \frac{1}{2\sqrt{n+1}}, \frac{1}{2})$; and (c) $b \in (0, \frac{1}{2} - \frac{1}{2\sqrt{n+1}})$. The game is illustrated in Table (5.2).

Table 5.2: Payoff Matrix

		Firm B: H	Firm B: L	
Firm A	H	$-c_A$, $-c_B$	$\pi_H^{HL}(c_A)$, $\pi_L^{HL}(c_B)$	q_A
	L	$\pi_L^{HL}(c_A)$, $\pi_H^{HL}(c_B)$	0, 0	$1 - q_A$
		q_B	$1 - q_B$	

[a] Firm $f, f \in \{A, B\}$ chooses from the clean technology H or the polluting technology L.
[b] The probability $q_f, f \in \{A, B\}$ is the probability that firm f chooses the clean technology H.

where the payoffs

$$\begin{cases} \pi_H^{HL} = \dfrac{n+1}{4} - c_f \text{ and } \pi_L^{HL} = 0, & \text{if } b \in [\dfrac{1}{2}, +\infty) \\ \pi_H^{HL} = (1+n)(1-b)b - c_f \text{ and } \pi_L^{HL} = 0, & \text{if } b \in [\dfrac{1}{2} - \dfrac{1}{2\sqrt{n+1}}, \dfrac{1}{2}) \\ \pi_H^{HL} = \dfrac{n}{4} - c_f \text{ and } \pi_L^{HL} = \dfrac{1}{4} - \dfrac{1}{2}b, & \text{if } b \in (0, \dfrac{1}{2} - \dfrac{1}{2\sqrt{n+1}}) \end{cases} \tag{5.17}$$

are generated by Equilibria *[EQU1]* - *[EQU3]*. c_f is the cost of investing in the clean technology for each firm, $f \in \{A, B\}$. $q_f \in [0, 1]$ is the probability that firm f chooses to invest.

We immediately see that there are two pure strategy Nash equilibria (H,L) and (L,H) which represent the cases where only one firm invests in the clean technology. These two equilibria therefore give us the mixed strategy Nash equilibrium. For Firm $f \in \{A, B\}$, the utility of choosing the clean technology H should be equal to that of choosing the polluting technology.

$$\begin{aligned} U_f(H) &:= (-c_{-f})q_{-f} + \pi_H^{HL}(c_f)(1-q_{-f}) \\ &= \pi_L^{HL}(c_f)q_{-f} + 0 =: U_f(H) \\ \Rightarrow q_f &= \frac{\pi_H^{HL}(c_{-f})}{(c_{-f}) + \pi_H^{HL}(c_{-f}) + \pi_L^{HL}(c_{-f})} \end{aligned} \tag{5.18}$$

Therefore we derive the subgame perfect equilibria in the game of boycott.

Proposition 13. *$\forall b$, there is a equilibrium in the boycott game such that two firms choose different production technology in the first stage and choose the pricing strategies in the second stage as shown in [EQU1] - [EQU3]. The payoffs are given by (5.17); [SPE1]*

$\forall b$, there is a equilibrium in the boycott game such that firm $f \in \{A, B\}$ chooses the clean technology with probability q_f that is given by Equation (5.18) in the first stage and chooses the pricing strategy in the second stage as shown in [EQU4] - [EQU5]. [SPE2]

The results suggest that it is profitable for a firm to differentiate itself by investing in expensive clean technology, supported by non-zero profits in the pure strategy equilibria. If two firms use symmetric mixed strategies, the clean technology would be adopted with some probability that is jointly determined by the cost of investing (c), the number of a-type consumers with high moral concern (n) and the cost of social pressure (b).

5.6 Conclusion

We discuss the Glazer et al. (2010) and suggest the existence of other equilibria in the output determining stage. We demonstrate our argument by constructing the equilibria in two polar cases. We then derive the subgame perfect equilibria for the whole game.

As an extension to the original article, we consider the Bertrand competition. The game is solved by backward induction. In the second stage, we derive the optimal pricing strategies considering firms' technology selection made in the previous stage: (1) only one firm invests in clean technology; (2) both firms invest; and (3) no firm invests. We find that, in the first case, the ethical firm using the expensive environment-friendly technology (Firm H) is able to make non-zero profit at equilibria due to the consumers' different preferences and the benefit arising from the cost of social pressure. The non-ethical firm using the cheap polluting technology (Firm L) is able to make non-zero profit at equilibrium only if b is sufficiently small. In the second and third cases, it becomes a standard price competition model where both firms make zero revenue by setting the same price at zero.

The pricing strategies that generates positive profits are highlighted as follows:

(a) When $b \in [\frac{1}{2}, +\infty)$, the ethical firm with the clean technology is able to make non-zero profit if the cost of investing in the clean technology c is small enough and n is large enough such that $\pi_H^{HL}(P_H = \frac{1}{2}) = \frac{n}{4} - c > 0$;

(b) When $b \in [\frac{1}{2} - \frac{1}{2\sqrt{n+1}}, \frac{1}{2})$, the ethical firm is able to make non-zero profit if c is small enough and n is large enough such that $\pi_H^{HL}(P_H = b) = (1+n)(1-b)b - c > 0$;

(c) When $b \in (0, \frac{1}{2} - \frac{1}{2\sqrt{n+1}})$, by assuming that the consumer who is indifferent between two firms chooses L firm, both firms are able to make non-zero profit for suitable c and n such that $\pi_H^{HL}(P_H = \frac{1}{2}) = \frac{n}{4} - c > 0$ and $\pi_L^{HL}(P_L = \frac{1}{2} - b) = \frac{1}{4} - \frac{1}{2}b > 0$.

The results suggest that firms' strategies largely depend on the value of social cost b. In a market where not many consumers care about environmental protection, firms prefer to use the polluting technology. This implies that improving society's awareness of social responsibility increases social pressure on non-ethical attitudes and behaviour. Thus consumers' incentives to boycott grow stronger. A higher boycotting power leads to a higher probability to force the firms to act ethically.

We then solve the strategies in production technology selection in the first stage. Therefore we derive the subgame perfect equilibria of the boycott game. We conclude that investing in the clean technology (behaving ethically) is not necessarily the optimal strategy for firms, in terms of payoff maximizing, although the ethical firms do benefit from the consumers' moral concerns and the cost of social pressure.

Bibliography

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